Voting in Participatory Budgeting: A Survey

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What is Participatory Budgeting (PB)?

Origins

- Most generally: Letting citizens have a say how government spends its money.
- Emerged in 1990s in Brasil, then spread through South America. Most commonly via discussion and deliberation in neighborhood plenary meetings.
- Used on level of cities, schools, housing complexes.

Yves Cabannes. "Participatory budgeting: a significant contribution to participatory democracy". In: *Environment and Urbanization* 16.1 (2004), pp. 27–46

Note that this paper does not ever mention the possibility of *voting* within PB.

What is Participatory Budgeting (PB)?

Current PB in Europe and North America

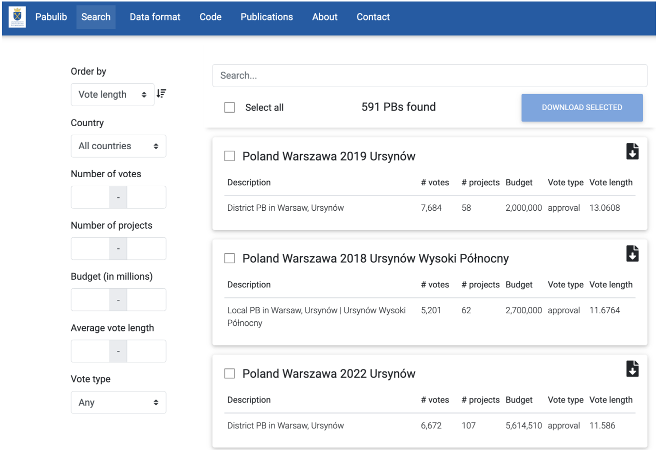
1. City government/parliament designates fixed budget for PB
2. Residents are invited to submit project proposals
3. City officials decide if projects are in scope, suggest changes, merge similar proposal, shortlist projects
4. Residents vote over projects (approval, online and/or paper)
5. Greedily take projects with highest score until budget runs out
6. City officials oversee implementation

Haris Aziz and Nisarg Shah. "Participatory Budgeting: Models and Approaches". In: *Pathways between Social Science and Computational Social Science: Theories, Methods and Interpretations*. Ed. by T. Rudas and P. Gábor. Springer, 2020

Some PB Implementations

City	Years	Budget	Method
Madrid	2018–19	EUR 100m	Knapsack votes, city+district
Madrid	2022	EUR 50m	Knapsack votes, city+district
Barcelona	2021	EUR 30m	Knapsack votes, 2 districts
Paris	2016–19	EUR 100m	4-approval, city+district
Paris	2021–22	EUR 75m	Majority judgment, unit cost
Lyon	2022	EUR 12.5m	10-approval
Strasbourg	2021	EUR 2m	Distribute 5 points to projects
Cambridge MA	2015–22	USD 1m	5-approval
New York City	2015–19	USD 40m	5-approval
Montreal	2021	CAD 25m	5-approval
Reykjavík	2021	IKR 850m	Knapsack vote+ "star" 1 project
Warsaw	2016–19	PLN 65m	Knapsack votes, city+district
Warsaw	2020–22	PLN 85m	10-approval, 15-approval district
Częstochowa	2022	PLN 10m	Distribute 10 points to projects
Gdansk	2022	PLN 20m	Distribute 5 points to projects
Krakow	2022	PLN 28m	Rank 3 projects, Borda scores
Portugal	2018	EUR 5m	1 vote nation-wide, 1 vote region

Pabulib



Dariusz Stolicki, Stanislaw Szufa, and Nimrod Talmon. "Pabulib: A Participatory Budgeting Library". In: *arXiv:2012.06539* (2020)

Basic Model

- Let $N = \{1, \ldots, n\}$ be the set of voters.
- Let $C = \{c_1, \ldots, c_m\}$ be the set of projects.
- Each project c_j has a cost: $\mathrm{cost}(c_j) \geq 0$.
  - For $T \subseteq C$, write $\mathrm{cost}(T) = \sum_{c \in T} \mathrm{cost}(c)$.
- Budget limit $B \geq 0$.
- An outcome is a set $W \subseteq C$ that is affordable: $\mathrm{cost}(W) \leq B$.
- Additive utilities: each voter $i \in N$ has utilities $u_i(c) \geq 0$ for all the projects $c \in C$, and the utility of a set of projects is the sum $u_i(T) = \sum_{c \in T} u_i(c)$.

Could be enriched with additional feasibility constraints, negative utilities, non-additive utilities, ...

- If $\mathrm{cost}(c) = 1$ for all $c \in C$, and $B \in \mathbb{N}$, we are in the unit cost case $\rightarrow$ committee elections

Approval Ballots

In almost all implementations, approval ballots are used.

— Ballot Paper —

Total available budget: € 3 000 000.

Approve up to 4 projects.

- [x] Extension of the Public Library
  Cost: € 200 000
- [ ] Photovoltaic Panels on City Buildings
  Cost: € 150 000
- [x] Bicycle Racks on Main Street
  Cost: € 20 000
- [ ] Sports Equipment in the Park
  Cost: € 15 000
- [ ] Renovate Fountain in Market Square
  Cost: € 65 000
- [ ] Additional Public Toilets
  Cost: € 340 000
- [ ] Digital White Boards in Classrooms
  Cost: € 250 000
- [ ] Improve Accessibility of Town Hall
  Cost: € 600 000
- [x] Beautiful Night Lighting of Town Hall
  Cost: € 40 000
- [ ] Resurface Broad Street
  Cost: € 205 000

Approval Ballots

- An approval set of voter i is a subset $A_i \subseteq C$ of projects.
- The approval utilities induced by A_i are

$$u_i(c) = \begin{cases} 1 & \text{if } c \in A_i, \\ 0 & \text{if } c \notin A_i. \end{cases}$$

  - $u_i(W) = |W \cap A_i|$, the number of selected approved projects.

  Problem: Doesn't distinguish between cheap and expensive projects.
- The cost utilities induced by A_i are

$$u_i(c) = \begin{cases} \text{cost}(c) & \text{if } c \in A_i, \\ 0 & \text{if } c \notin A_i. \end{cases}$$

  - $u_i(W) = \text{cost}(W \cap A_i)$, the spending on approved projects.

  Problem: A project becomes more attractive if it is less efficient.

Utilitarian Approval Methods

- Maximize approval utilities
  - Optimum Knapsack: Select W maximizing $\sum_{i \in N} |A_i \cap W|$.
  - Greedy: Go through projects in order of approval score divided by cost, fund if possible else skip.
- Maximize cost utilities
  - Optimum: Select W maximizing $\sum_{i \in N} \mathrm{cost}(A_i \cap W)$.
  - Greedy: Go through projects in order of approval score, fund if possible else skip. $\leftarrow$ this is the one used in practice!

Nimrod Talmon and Piotr Faliszewski. "A framework for approval-based budgeting methods". In: *Proceedings of the 33rd AAAI Conference on Artificial Intelligence (AAAI)*. 2019, pp. 2181–2188

Ashish Goel et al. "Knapsack Voting for Participatory Budgeting". In: *ACM Transactions on Economics and Computation* 7.2 (2019), 8:1–8:27

Federica Ceron, Stéphane Gonzalez, and Adriana Navarro-Ramos. "Axiomatic characterizations of the knapsack and greedy participatory budgeting methods". In: (2022). Working Paper

Is there a better name than "greedy"?

What to do about districts?

- Almost all cities pre-divide the budget among districts (proportional to population) to hold separate district elections
- Voters are allowed to choose 1 district and vote only there
  - In Gdansk can vote on all projects
  - Some cities allow you to vote in 2 districts
  - Often there is also an election for global projects
- Problem: underfund projects of interest to several districts

> D. Ellis Hershkowitz et al. “District-fair participatory budgeting”. In: *Proceedings of the 35th AAAI Conference on Artificial Intelligence*. 2021, pp. 5464–5471

- Possible solution: Allow voters to vote globally. Run virtual district elections, see how much utility each district would get. Then add constraints to the global election saying that each district should get at least that much.
- Probably better solution: give guarantees to all groups with similar interests

Method of Equal Shares

Dominik Peters, Grzegorz Pierczyński, and Piotr Skowron. "Proportional participatory budgeting with additive utilities". In: *Proceedings of the 35th Conference on Neural Information Processing Systems (NeurIPS)*. 2021, pp. 12726–12737

There is a simple, polynomial time method that enjoys strong proportionality properties.

- Equally split the budget between voters.
- Look for project whose approvers can pay for it.
- Buy project, and share the cost equally between its approvers.
- If there are several options, take the one where we can spread the cost most thinly.

It works for approval but extends to additive utilities.

Method of Equal Shares: Example

Budget $B = \$100$, 10 voters, everyone starts with \$10.

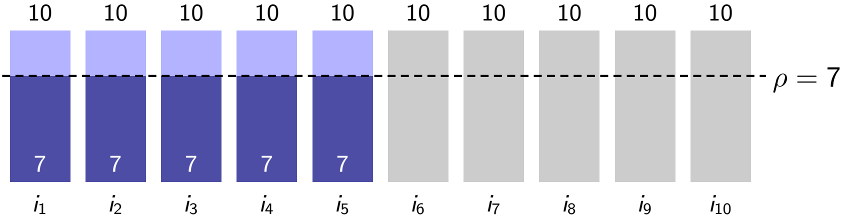
	cost	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
P1	\$36	1	1	1	1	1	0	0	0	0	0
P2	\$36	1	1	1	1	1	1	0	0	0	0
P3	\$25	1	1	1	1	1	0	0	0	0	0
P4	\$24	0	0	0	1	1	1	1	0	0	0
P5	\$24	0	0	0	0	0	1	1	1	1	0

Equal Shares will select Project 3, then Project 5, then terminate.

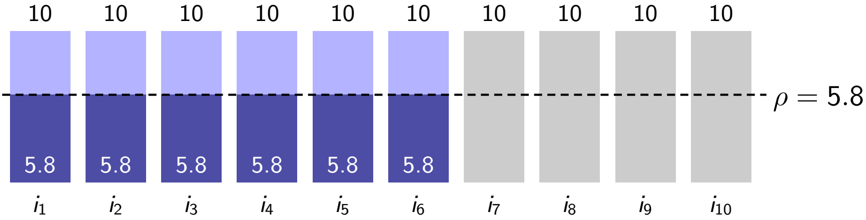
Method of Equal Shares: More Popular is Better

Budget $B = \$100$, 10 voters, everyone starts with \$10.

Project 1 with cost \$35



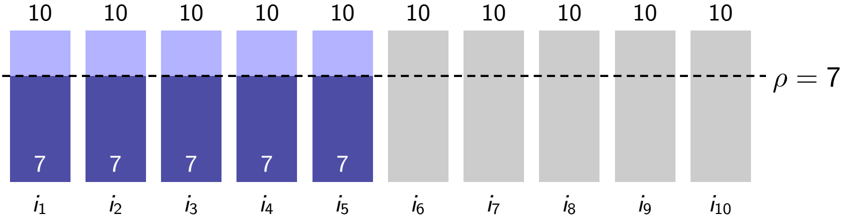
Project 2 with cost \$35 but more approvers



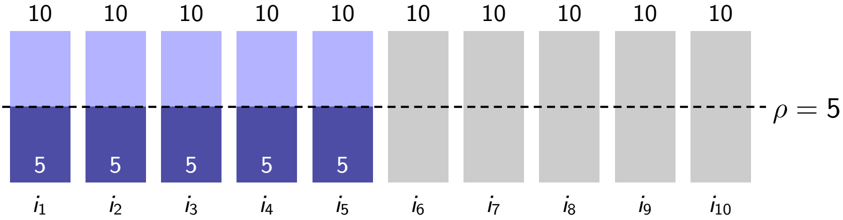
Method of Equal Shares: Cheaper is Better

Budget $B = \$100$, 10 voters, everyone starts with \$10.

Project 1 with cost \$35

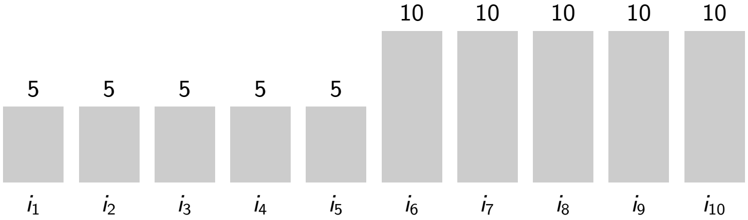


Project 3 with cost \$25 with same approvers



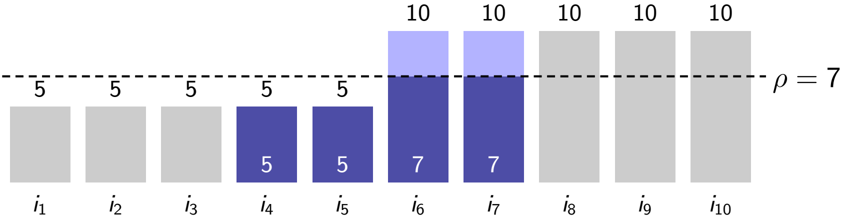
Method of Equal Shares

Implement Project 3, so $i_1, \ldots, i_6$ each spend \$5.

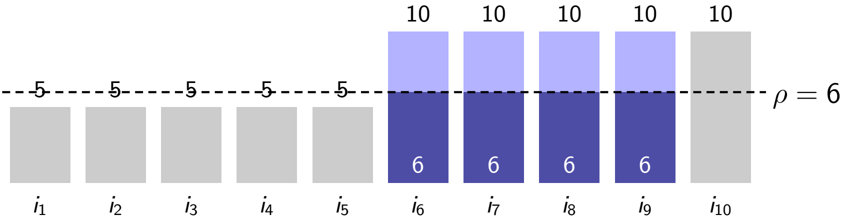


Method of Equal Shares: Richer is Better

Project 4 with cost \$24



Project 5 with cost \$24 but with richer approvers



Method of Equal Shares: Completions

In the example, $B = \$100$, but Equal Shares only spends $\$25 + \$24 = \$49$! Unselected projects 1, 2, and 4 each would still fit into the budget limit. (Equal Shares is not "exhaustive".)

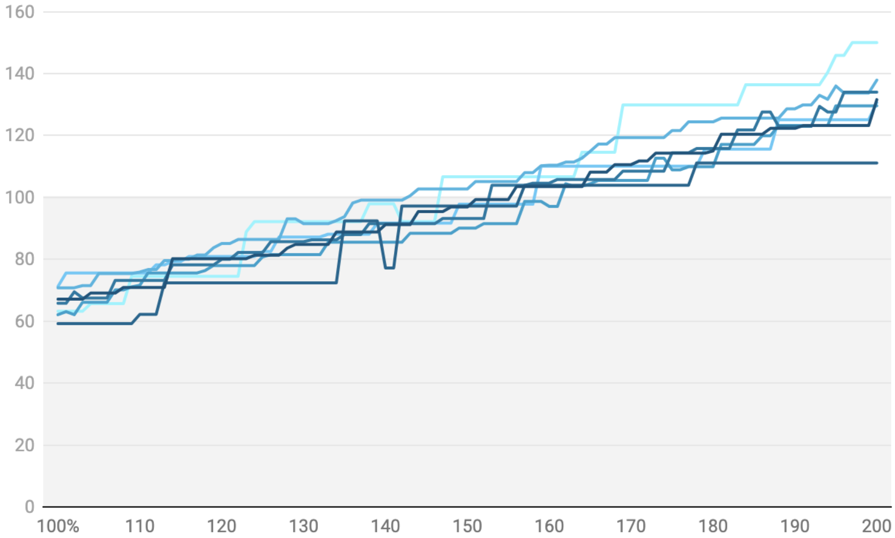
Intuition: Equal Shares only spends money if fairness forces it to.

Completion strategies:

- Completion by varying the budget: repeatedly increase the budget by \$1 until the next increment would make the output infeasible.
- Completion by utilitarian: continue using standard greedy.
- Other proposals: completion by Phragmén's method, completion by perturbation.
- Do not complete, save money (maybe for next year).

Method of Equal Shares: Varying the budget

Pabulib data, Warsaw 2021 election. Each line = one district.
x-axis: input budget. y-axis: cost of Equal Shares outcome



Equal Shares satisfies EJR

Theorem

For approval utilities, the outcome of Equal Shares satisfies *Extended Justified Representation:*

> *If $S \subseteq N$ is a group of voters, and $T \subseteq C$ a proposal that*
> - *S can afford: $\text{cost}(T) \leq B \cdot |S|/|N|$, and*
> - *S unanimously approves: $T \subseteq \bigcap_{i \in S} A_i$,*
>
> *then at least 1 voter in S approves at least $|T|$ projects in the Equal Shares outcome, so $u_i(W) \geq u_i(T)$.*

- The only natural method known to satisfy EJR.
- *Proof idea*: The projects in set T offer good "bang per buck" to S. While voters have not reached their budget limit, Equal Shares always selects the projects with the best bang per buck. Consider the first agent in S whose money runs out: that agent spent her money so effectively that she gets enough utility.

Method of Equal Shares for additive utilities

Equal Shares can also be defined for general additive valuations.

Idea: voters contribute to the cost of a project in proportion to their utility, so voter i pays $\rho \cdot u_i(c)$ for c, for some ρ such that

$$\sum_{i \in N} \min\{\rho \cdot u_i(c),\ i\text{'s remaining budget}\} = \text{cost}(c).$$

If a project has smaller ρ then it offers better bang-per-buck.

Equal Shares at each step selects a project with minimum ρ.

Theorem

For additive utilities, Equal Shares satisfies Extended Justified Representation up to 1 project (EJR1).

- For general additive utilities, satisfying EJR is weakly NP-hard.
- But there is an existence proof for EJR.

Method of Equal Shares and EJR

	Approval utilities	Additive utilities
Unit costs	EJR	EJR up to one project
General costs	EJR	EJR up to one project†

$\dagger$: Unless P $=$ NP, no strongly polynomial time method (such as Equal Shares) can satisfy EJR.

Method of Equal Shares: implementation pseudocode

$W \leftarrow \emptyset$.
For each voter $i \in N$, $b[i] \leftarrow b/|N|$
while *true* **do**
  for $c \in C \setminus W$ **do**
    if $\sum_{i \in N : u_i(c) > 0} b[i] < \mathrm{cost}(c)$ **then**
      $\rho(c) \leftarrow \infty$ (project c is not affordable)
    else
      Let $i_1, \dots, i_t$ be a list of all voters $i \in N$ with $u_i(c) > 0$, ordered so that $b[i_1]/u_{i_1}(c) \leq \dots \leq b[i_t]/u_{i_t}(c)$.
      for $s = 1, \dots, t$ **do**
        $\rho(c) \leftarrow (\mathrm{cost}(c) - (b[i_1] + \dots + b[i_{s-1}]))/(u_{i_s}(c) + \dots + u_{i_t}(c))$
        if $\rho(c) \cdot u_{i_s} \leq b[i_s]$ **then**
          break (we have found the ρ-value)
  if $\min_{c \in C \setminus W} \rho(c) = \infty$ **then**
    return W
  $c \leftarrow \mathrm{argmin}_{c \in C \setminus W} \rho(c)$ (break ties arbitrarily)
  $W \leftarrow W \cup \{c\}$
  for $i \in N$ *such that* $u_i(c) > 0$ **do**
    $b[i] \leftarrow b[i] - \min\{b[i], \rho(c) \cdot u_i(c)\}$

Method of Equal Shares for cost utilities

Standard approval-based (0/1) Equal Shares selects projects in order of (# of votes)/cost.
But most cities want to go by just # of votes.

Solution: use cost utilities with Equal Shares.

Simple explanation of Equal Shares with cost utilities:

- Repeatedly select project with highest number of votes, provided its supporters have enough money for it.
- Split the cost equally among the supporters.
- If a voter runs out of money, delete the voter and update the vote counts.*

* if a voter has very little money left, the voter will only count fractionally when calculating vote counts.

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Method of Equal Shares

From Wikipedia, the free encyclopedia

The Method of Equal Shares[1][2][3][4] (in early papers the method has been also referred to as **Rule X**,[2][3][4] but since 2022 the authors started using the name "method of equal shares"[1]) is a proportional method of counting ballots that applies to participatory budgeting[1] to committee elections[2] and to simultaneous public decisions.[5][3] It can be used, when the voters vote via approval ballots, ranked ballots or cardinal ballots.

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Electoral systems

Single-winner/majoritarian [show]
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Politics portal

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Other rules

- Phragmén's Method. Voters start with \$0 each. Give each voter \$1 per second. Once there is a project whose supporters have enough money to buy it, stop. Buy the project, and reset the balance of the supporters to \$0. Continue.
  - Fails EJR, and doesn't extend beyond approval.
  - Seems pretty good in practice.
- Thiele's Method / Proportional Approval Voting (PAV). Similar to maximizing Nash welfare. Works great for unit costs. Fails proportionality badly otherwise.

Maaike Los, Zoé Christoff, and Davide Grossi. "Proportional Budget Allocations: A Systematization". In: *IJCAI 2022. arXiv:2203.12324* (2022)

Haris Aziz, Barton E. Lee, and Nimrod Talmon. "Proportionally Representative Participatory Budgeting: Axioms and Algorithms". In: *Proceedings of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*. 2018, pp. 23–31

Dominik Peters, Grzegorz Pierczyński, and Piotr Skowron. "Proportional participatory budgeting with additive utilities". In: *Proceedings of the 35th Conference on Neural Information Processing Systems (NeurIPS)*. 2021, pp. 12726–12737

Core

Let $S \subseteq N$ be a coalition of voters.
Let $T \subseteq C$ be a proposal of projects that

- can be afforded by S, so $\text{cost}(T)/B \leq |S|/|N|$
- ~~is unanimously approved by S, so $T \subseteq A_i$ for all $i \in S$.~~

Then there exists $i \in S$ with $u_i(W) \geq u_i(T)$.

- Open Question: For approval utilities does there always exist a core outcome?
  For 0/1/2 utilities it can be empty, even with unit costs (Condorcet cycle).
  I don't know if people have thought about cost utilities.

Core Approximations

- Utility approximation
  - "can't have a deviation T where each member of S more than doubles utility"
  - Equal Shares approximates within $O(\log(|W|))$.
  - Rounding fractional core: Existence of 9.27-approximation.

Dominik Peters, Grzegorz Pierczyński, and Piotr Skowron. "Proportional participatory budgeting with additive utilities". In: *Proceedings of the 35th Conference on Neural Information Processing Systems (NeurIPS)*. 2021, pp. 12726–12737

Kamesh Munagala et al. "Approximate core for committee selection via multilinear extension and market clearing". In: *Proceedings of the 2022 Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*. SIAM. 2022, pp. 2229–2252

Brandon Fain, Kamesh Munagala, and Nisarg Shah. "Fair Allocation of Indivisible Public Goods". In: *Proceedings of the 19th ACM Conference on Economics and Computation (ACM-EC)*. 2018, pp. 575–592

Core Approximations

- Entitlement approximation
  - “can’t have a deviation T that is twice cheaper than what S is allowed to deviate with”
  - equivalent: “can find a core outcome if we are allowed to overspend by a factor of 2“
  - Always exists a 32-approximation.
  - Conjecture: 2-approximation (which would be tight).

Zhihao Jiang, Kamesh Munagala, and Kangning Wang. “Approximately stable committee selection”. In: *Proceedings of the 52nd Annual ACM SIGACT Symposium on Theory of Computing*. 2020, pp. 463–472

Strategyproofness

Dominik Peters. "Proportionality and Strategyproofness in Multiwinner Elections". In: *Proceedings of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*. vol. 1549–1557. 2018

Boas Kluiving et al. "Analysing irresolute multiwinner voting rules with approval ballots via sat solving". In: *Proceedings of the 24th European Conference on Artificial Intelligence (ECAI)*. 2020, pp. 131–138

- Proportional rules must be manipulable.
- Voters can pretend to not approve popular projects.
- With unit costs, the greedy method is strategyproof, but not true in PB.
- If we are allowed to implement the last project fractionally, knapsack voting is strategyproof under a type of cost utilities.

Ashish Goel et al. "Knapsack Voting for Participatory Budgeting". In: *ACM Transactions on Economics and Computation* 7.2 (2019), 8:1–8:27

- Additive valuations: strategyproofness mostly impossible.

Eric Bahel and Yves Sprumont. "Strategy-proof choice with monotonic additive preferences". In: *Games and Economic Behavior* 126 (2021), pp. 94–99

Computational Complexity

- Utilitarian welfare maximization $\iff$ Knapsack
- Maximize Nash welfare ($\sum_{i\in N} \log(u_i(W)+1)$) NP-hard, even for single-peaked / single-crossing / few voters (W[1])
- "Chamberlin–Courant" ($\sum_{i\in N} \max_{c\in W} u_i(c)$) NP-hard but easy for single-peaked / single-crossing / few voters (FPT)

Till Fluschnik et al. "Fair knapsack". In: *Proceedings of the 33rd AAAI Conference on Artificial Intelligence (AAAI)*. 2019, pp. 1941–1948

- Hardness also often appears for utilitarian welfare when imposing additional constraints, or when allowing substitutes and complementarities.

Pallavi Jain et al. "Participatory Budgeting with Project Groups". In: *Proceedings of the 30th International Joint Conference on Artificial Intelligence (IJCAI)*. 2021, pp. 276–282

Pallavi Jain, Nimrod Talmon, and Laurent Bulteau. "Partition aggregation for participatory budgeting". In: *Proceedings of the 20th International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*. 2021, pp. 665–673

Extensions I

- JR and EJR1? EJR in pseudo-polynomial time? FJR?
- Accountability/transparency: how to "prove" to the public that we calculated the outcome correctly?
- Understand voting data: how to identify groups with similar interests?
- Negative votes: Allow voters to vote against a project. Some might not want a particular project to be implemented near them. Also useful for non-PB applications of the same model, e.g. allowing downvotes.
- "At most one of these" constraints: Empty plot of land, many projects that could be implemented there. Equal Shares has a natural generalization but does not satisfy EJR anymore.
  - A solution would work for proportional multi-issue elections.

Piotr Skowron and Adrian Górecki. "Proportional Public Decisions". In: *Proceedings of the 36th AAAI Conference on Artificial Intelligence*. 2022

Roy Fairstein, Reshef Meir, and Kobi Gal. "Proportional Participatory Budgeting with Substitute Projects". In: *arXiv:2106.05360* (2021)

Extensions II

- Arbitrary constraints: Allow arbitrary constraints on the collection of feasible sets. Perhaps go via judgment aggregation (JA). [And import proportionality to JA!]

> Simon Rey, Ulle Endriss, and Ronald de Haan. "Designing Participatory Budgeting Mechanisms Grounded in Judgment Aggregation". In: *Proceedings of the 17th International Conference on Principles of Knowledge Representation and Reasoning (KR)*. 2020

- Agenda setting and shortlisting: Very important step in PB is to decide what is the set of projects. PB officials merge projects and shortlist them (sometimes involving a citizen jury). Could be formally studied.

> Simon Rey, Ulle Endriss, and Ronald de Haan. "Shortlisting Rules and Incentives in an End-to-End Model for Participatory Budgeting". In: *Proceedings of the 30th International Joint Conference on Artificial Intelligence (IJCAI)*. 2021

Extensions III

- Input formats: How to best elicit preferences? Rankings, ratings, knapsack votes, 10-approval, value for money, ...

> Gerdus Benade et al. "Preference elicitation for participatory budgeting". In: *Management Science* 67.5 (2021), pp. 2813–2827

> Haris Aziz and Barton E. Lee. "Proportionally representative participatory budgeting with ordinal preferences". In: *Proceedings of the 35th AAAI Conference on Artificial Intelligence (AAAI)*. 2021, pp. 5110–5118

> Piotr Skowron et al. "Participatory budgeting with cumulative votes". In: *arXiv:2009.02690* (2020)

- Time: PB is often repeated every year. We might want to be fair to people/groups over the long term, or we could allow people to invest or to save money for future years.

> Martin Lackner, Jan Maly, and Simon Rey. "Fairness in long-term participatory budgeting". In: *Proceedings of the 20th International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*. 2021, pp. 1566–1568

Extensions IV

- Allow for some divisible projects. Milestones.
- Multiknapsack. Allow for several budget constraints simultaneously, such as money and time and CO2e emissions.
- Separate budgets. In practice, voters vote for several budgets at the same time (city-wide, district) with disjoint project proposals. Is there something better than running the same voting rule separately for each?
- Agent contributions: Allow agents to add their own money to the budget.

Haris Aziz and Aditya Ganguly. "Participatory Funding Coordination: Model, Axioms and Rules". In: *Proceedings of the 7th International Conference on Algorithmic Decision Theory (ADT)*. 2021, pp. 409–423

Jiehua Chen, Martin Lackner, and Jan Maly. "Participatory Budgeting with Donations and Diversity Constraints". In: *Proceedings of the 36th AAAI Conference on Artificial Intelligence (AAAI)*. 2022, pp. 9323–9330