Generalizing Instant Runoff Voting to Allow Indifferences

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Introduction

Instant Runoff Voting (IRV) is a widely used single-winner voting rule based on (possibly truncated) linear orders. Many electoral reform advocates prefer this method.

Alaska / Maine / NYC / SF / . . .   Australia (since 1918)   Ireland

This method repeatedly eliminates the candidate that is ranked top least often, until only one candidate remains who is the winner.

We ask: what is the right way to generalize IRV to weak orders (allowing indifferences)?

We consider two natural options:

- Approval-IRV
- Split-IRV

Approval-IRV

We propose Approval-IRV. At each step, interpret each vote as an approval vote for its top-ranked uneliminated alternatives. Delete the candidate with the fewest approvals.

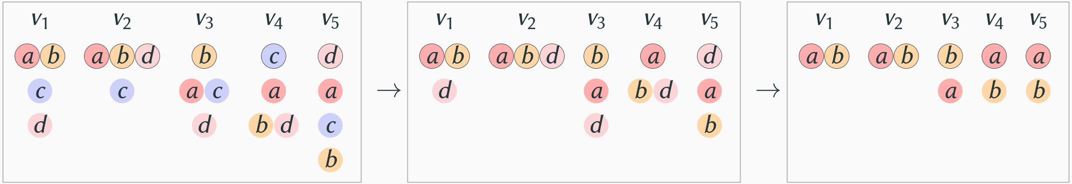


Figure 1: An example of Approval-IRV with voters $v_1, \dots, v_5$. The first eliminated alternative is c, which is ranked on top only once. Then d is eliminated, and finally a wins the majority vote against b. Thus, a is the winner.

Alternative method: Split-IRV where a vote with 3 top-ranked alternatives gives $\frac{1}{3}$ points to these alternatives, and the lowest-scoring alternative is deleted. Equivalent to running IRV after the replacement operation.

History

Multiwinner Split-STV was developed in series of articles in the journal *Voting Matters*.

> Brian L. Meek. "A new approach to the Single Transferable Vote. Paper II: The problem of non-transferable votes". In: *Voting matters* (1 1994). URL: https://www.votingmatters.org.uk/issue1/P2.htm

> C. Hugh E. Warren. "STV and equality of preference". In: *Voting matters* (7 1996). URL: https://www.votingmatters.org.uk/issue7/P5.htm

Split-STV "was first used by the John Muir Trust (for Trustee elections) in 1998, and by the London Mathematical Society in 1999" and both still use Split-STV today

> Denis Mollison. "Fair votes in practice". In: *arXiv:2303.15310* (2023). URL: https://arxiv.org/abs/2303.15310

The only previous scholarly discussion of Approval-STV is by Janson (2016).

> Svante Janson. "Phragmén's and Thiele's election methods". In: *arXiv:1611.08826* (2016). URL: https://arxiv.org/abs/1611.08826

Since about 1996, there have been sporadic discussions of Approval-IRV on internet forums, see e.g., the election-methods mailing list (1996, 2004), electowiki, and reddit (2019). A 2004 webtool implements both Approval-IRV and Split-IRV.

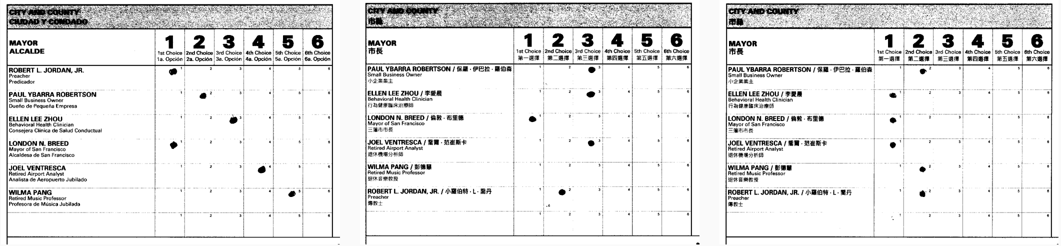
Reasons for using Weak Orders

- *A compromise between Ranked Choice Voting and Approval Voting.* FairVote FOR A MORE PERFECT UNION · THE CENTER FOR Election Science
- *More expressive,* when voters have true indifferences. (Australia forces no indifferences.)
- *More expressive,* when there are many candidates but at most 5 ranks on the ballot.
- *Less effort,* especially true for preferences like $a \succ b \succ \{c, d, e, f\} \succ g$.
- *Better alignment with candidate campaigns,* which typically only ask to be ranked #1, for example in NYC.

  > Lindsey Cormack. "More choices, more problems? Ranked choice voting errors in New York City". In: *American Politics Research* (2023). URL: https://dominik-peters.de/archive/cormack2023.pdf

- *Reduce need for some types of strategic voting.*

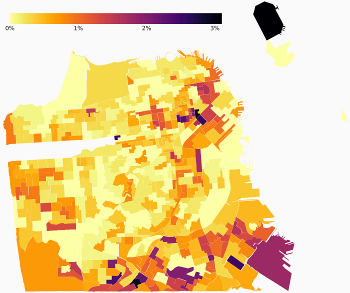
  > Alex Small. "Geometric construction of voting methods that protect voters' first choices". In: *arXiv:1008.4331* (2010). URL: https://arxiv.org/abs/1008.4331

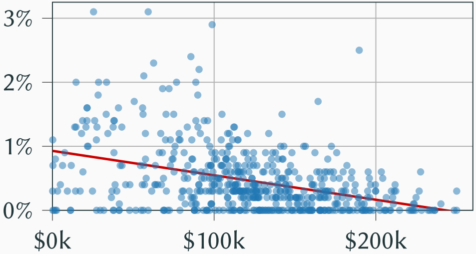


(a) Two top choices **(b)** Vetoing a candidate **(c)** An approval vote

Figure 2: Examples of ballots that can be interpreted as weak orders (2019 mayoral election in San Francisco).

SF: Locations of weak order ballots





(a) Map of San Francisco election precincts, colored by the fraction of votes that could be interpreted as a weak order with indifferences.

(b) Among election precincts, median household income (horizontal axis) is negatively correlated with percent of ballots showing a weak order (vertical axis; $r = -0.4$, $p < 0.001$).

Figure 3: Ballot data from the 2019 mayoral election in San Francisco.

Contribution

- Compare Approval-IRV and Split-IRV axiomatically.
  - Independence of Clones
  - Majority condition
  - Monotonicity
- Characterize Approval-IRV as the “right” generalization.
- Multi-winner Approval-STV preserves proportionality axioms.
- Experiments.

Independence of Clones

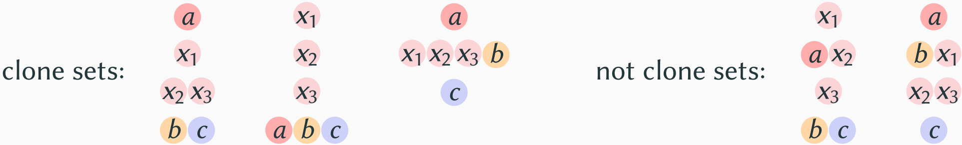


Figure 4: Examples of $X = \{x_1, x_2, x_3\}$ being a clone set or not being a clone set.

Definition

Independence of clones requires that when we add clones x_1 x_2 x_3 of a candidate x,

1. non-clones a b c are not affected (they win iff they previously won), and
2. if x was a winner, then one of its clones x_1 x_2 x_3 is a winner.

T. Nicolaus Tideman. “Independence of clones as a criterion for voting rules”. In: *Social Choice and Welfare* 4 (1987), pp. 185–206. DOI: 10.1007/BF00433944. URL: https://www.condorcet.vote/view/DOCS/IndependenceofClones.pdf

Independence of Clones

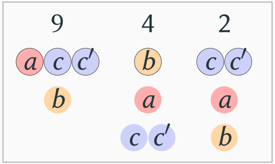


Figure 5: Split-IRV fails independence of clones.

Theorem

Approval-IRV is independent of clones.

Argument similar to linear-order version.

We give a rigorous proof by induction; also shows that linear-order IRV satisfies independence of clones.

Majority Condition

47%	4%	25%	24%
a b	a	c	d
c	b	b	b
d	c	d	c
	d	a	a

Figure 6: A problem with electing majority alternatives.

Linear-order IRV satisfies the *majority criterion*: if a majority of voters places a in top position, then a wins.

How to generalize to weak orders? Maybe "if some candidate is ranked top by a majority, then such a candidate should win"?

In the figure, this implies a is the winner.
But 49% say $b \succ a$ and only 4% say $a \succ b$.

Bad axiom! Need a different generalization.

Approval-IRV: b (also Condorcet winner)
Split-IRV: a

Respect for Cohesive Majorities

9	5	5	8	10
a b c	a b	a c	b c d	d
d	d	d	a	a b c
	c	b		

Figure 7: Split-IRV violates respect for cohesive majorities because it eliminates a, then b and c, and elects d.

Respect for cohesive majorities:
If a majority of voters rank c on top ("cohesive") then the winner must be ranked top by at least one member of that majority.

Theorem

Approval-IRV respects cohesive majorities.

Characterization within Elimination Scoring Rules

Theorem

Approval-IRV is the only elimination scoring rule satisfying independence of clones and respect for cohesive majorities.

The axioms are independent.

An *elimination scoring rule* sequentially eliminates the lowest-scoring candidate, where the scores are positional scores (weakly decreasing) that may be different for each *order type* τ (specifying the sizes of the indifference classes).

Examples: different versions of Borda scoring
Approval: $\tau \mapsto (1, 0, \ldots, 0)$
Split: $\tau \mapsto (1/\tau_1, 0, \ldots, 0)$.

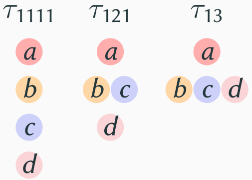
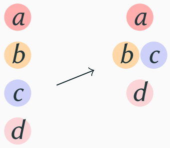


Figure 8: Examples of weak orders with different order types.

John H. Smith. "Aggregation of preferences with variable electorate". In: *Econometrica: Journal of the Econometric Society* 41.6 (1973), pp. 1027–1041. DOI: 10.2307/1914033

Indifference Monotonicity



A c -*hover* is the following type of transformation:

$$\begin{aligned} & C_1 \succ \cdots \succ C_j \succ \{c\} \succ C_{j+2} \succ \cdots \succ C_k \\ \longmapsto\ & C_1 \succ \cdots \succ C_j \cup \{c\} \succ C_{j+2} \succ \cdots \succ C_k \end{aligned}$$

Note: c must initially lie in a singleton indifference class.

Definition (Indifference monotonicity)

If $c \in f(P)$ is a winner and we apply some c-hovers, then c remains a winner.

Characterization II within Elimination Scoring Rules

Theorem

Approval-IRV is the only elimination scoring rule that agrees with IRV on profiles of linear orders and that is indifference monotonic.

The axioms are independent.

Multiwinner Version: STV

We can define a weak-order version of the multi-winner rule STV, giving Approval-STV. STV gives proportional representation, which has been formalized via the Proportionality for Solid Coalition property.

It has been generalized to weak orders (PJR-style).

> Haris Aziz and Barton E. Lee. "The expanding approvals rule: improving proportional representation and monotonicity". In: *Social Choice and Welfare* 54 (2020), pp. 1–45. DOI: 10.1007/s00355-019-01208-3. URL: https://www.cse.unsw.edu.au/~haziz/prsolution.pdf

Definition (Generalized PSC, informal)

If a coalition of $\alpha\%$ of voters all agree that $T \succcurlyeq C \setminus T$, then at least $\alpha\%$ of the k winners should come from T (or equivalently liked candidates).

Theorem

Approval-STV satisfies generalized PSC for weak orders.

Proof of some independent interest also for the linear-order variant.

Experiments

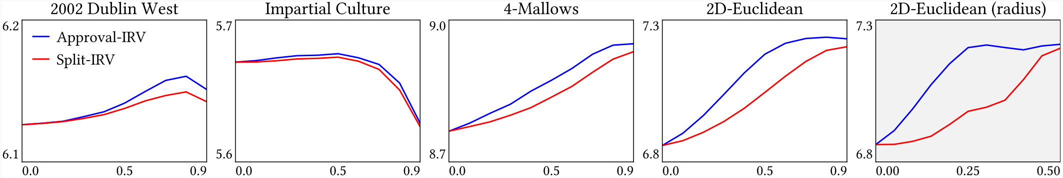


Figure 9: Average Borda score of the winner (normalized by dividing by n) for various datasets.

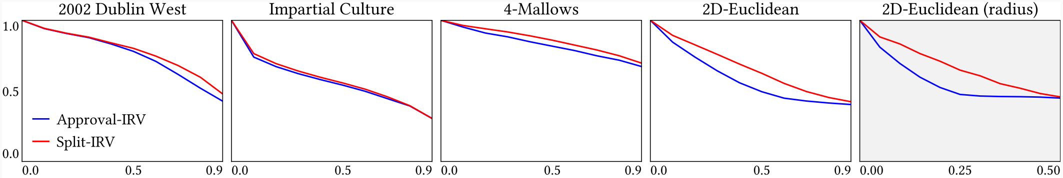


Figure 10: Frequency of agreement between the rule and linear-order IRV for various datasets.

x-axis: few indifferences $\longrightarrow$ *many indifferences*

Experiments

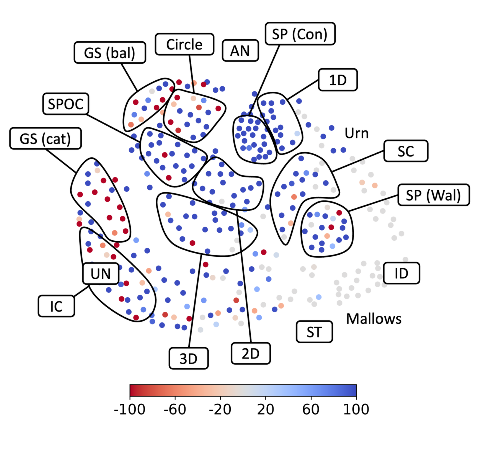


Figure 11: Map of elections, showing the difference in Borda score between the Approval-IRV and Split-IRV winner in the coin-flip model (blue: approval better than split).

Niclas Boehmer et al. “Understanding Distance Measures Among Elections”. In: *Proceedings of the 31st International Joint Conference on Artificial Intelligence (IJCAI).* 2022, pp. 102–108. DOI: 10.24963/ijcai.2022/15

Conclusion

	Approval-IRV	Split-IRV
Independence of clones	✓	✗
Respecting cohesive majorities	✓	✗
Indifference monotonicity	✓	✗
Quality of winner	+	○
Same winner as linear order	–	often
Generalized PSC	✓	✗

Table 1: Comparison of properties satisfied by the rules.