Generalizing Instant Runoff Voting to Allow Indifferences

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Instant Runoff Voting (IRV) is a widely used single-winner voting rule based on (possibly truncated) linear orders. Many electoral reform advocates prefer this method.

This method repeatedly eliminates the candidate that is ranked top least often, until only one candidate remains who is the winner.

We ask: what is the right way to generalize IRV to weak orders (allowing indifferences)?

We consider two natural options:

- Approval-IRV
- Split-IRV
We propose Approval-IRV. At each step, interpret each vote as an approval vote for its top-ranked uneliminated alternatives. Delete the candidate with the fewest approvals.

\[ \begin{array}{cccccc}
 v_1 & v_2 & v_3 & v_4 & v_5 \\
 a & b & a & b & d \\
 c & c & a & c & a \\
 d & d & b & d & c \\
\end{array} \]

\[ \begin{array}{cccccc}
 v_1 & v_2 & v_3 & v_4 & v_5 \\
 a & b & a & b & d \\
 c & c & a & c & a \\
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\end{array} \rightarrow \]

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 a & b & b & a & a \\
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\end{array} \rightarrow \]

**Figure 1:** An example of Approval-IRV with voters \( v_1, \ldots, v_5 \). The first eliminated alternative is \( c \), which is ranked on top only once. Then \( d \) is eliminated, and finally \( a \) wins the majority vote against \( b \). Thus, \( a \) is the winner.

Alternative method: **Split-IRV** where a vote with 3 top-ranked alternatives gives \( \frac{1}{3} \) points to these alternatives, and the lowest-scoring alternative is deleted. Equivalent to running IRV after the replacement operation.
History

Multiwinner Split-STV was developed in series of articles in the journal *Voting Matters*.


Split-STV “was first used by the John Muir Trust (for Trustee elections) in 1998, and by the London Mathematical Society in 1999” and both still use Split-STV today


The only previous scholarly discussion of Approval-STV is by Janson (2016).


Since about 1996, there have been sporadic discussions of Approval-IRV on internet forums, see e.g., the election-methods mailing list (1996, 2004), electowiki, and reddit (2019). A 2004 webtool implements both Approval-IRV and Split-IRV.
Reasons for using Weak Orders

- A compromise between Ranked Choice Voting and Approval Voting.
- More expressive, when voters have true indifferences. (Australia forces no indifferences.)
- More expressive, when there are many candidates but at most 5 ranks on the ballot.
- Less effort, especially true for preferences like \( a \succ b \succ \{c, d, e, f\} \succ g \).
- Better alignment with candidate campaigns, which typically only ask to be ranked #1, for example in NYC.


- Reduce need for some types of strategic voting.

<table>
<thead>
<tr>
<th>MAYOR ALCALDE</th>
<th>1 2 3 4 5 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROBERT L. JORDAN JR</td>
<td>☐ ☐ ☐ ☐ ☐ ☐</td>
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<tr>
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<td>☐ ☐ ☐ ☐ ☐ ☐</td>
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<td>WILMA PANG</td>
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- **(a)** Two top choices
- **(b)** Vetoing a candidate
- **(c)** An approval vote

**Figure 2:** Examples of ballots that can be interpreted as weak orders (2019 mayoral election in San Francisco).
(a) Map of San Francisco election precincts, colored by the fraction of votes that could be interpreted as a weak order with indifferences.

(b) Among election precincts, median household income (horizontal axis) is negatively correlated with percent of ballots showing a weak order (vertical axis; $r = -0.4$, $p < 0.001$).

**Figure 3:** Ballot data from the 2019 mayoral election in San Francisco.
Contribution

- Compare Approval-IRV and Split-IRV axiomatically.
  - Independence of Clones
  - Majority condition
  - Monotonicity
- Characterize Approval-IRV as the “right” generalization.
- Multi-winner Approval-STV preserves proportionality axioms.
- Experiments.
Independence of Clones

Figure 4: Examples of $X = \{x_1, x_2, x_3\}$ being a clone set or not being a clone set.

Definition

*Independence of clones* requires that when we add clones $x_1 x_2 x_3$ of a candidate $x$,

1. non-clones $a \ b \ c$ are not affected (they win iff they previously won), and
2. if $x$ was a winner, then one of its clones $x_1 x_2 x_3$ is a winner.

Theorem

Approval-IRV is independent of clones.

Argument similar to linear-order version.

We give a rigorous proof by induction; also shows that linear-order IRV satisfies independence of clones.
Linear-order IRV satisfies the *majority criterion*: if a majority of voters places $a$ in top position, then $a$ wins.

How to generalize to weak orders? Maybe “if some candidate is ranked top by a majority, then such a candidate should win”?

In the figure, this implies $a$ is the winner.

But 49% say $b \succ a$ and only 4% say $a \succ b$.

Bad axiom! Need a different generalization.

Approval-IRV: $b$ (also Condorcet winner)

Split-IRV: $a$
Respect for Cohesive Majorities

Figure 7: Split-IRV violates respect for cohesive majorities because it eliminates \(a\), then \(b\) and \(c\), and elects \(d\).

*Respect for cohesive majorities:*

If a majority of voters rank \(c\) on top ("cohesive") then the winner must be ranked top by at least one member of that majority.

**Theorem**

*Approval-IRV respects cohesive majorities.*
Theorem

Approval-IRV is the only elimination scoring rule satisfying independence of clones and respect for cohesive majorities.

The axioms are independent.

An elimination scoring rule sequentially eliminates the lowest-scoring candidate, where the scores are positional scores (weakly decreasing) that may be different for each order type $\tau$ (specifying the sizes of the indifference classes).

Examples: different versions of Borda scoring
Approval: $\tau \mapsto (1, 0, \ldots, 0)$
Split: $\tau \mapsto (1/\tau_1, 0, \ldots, 0)$.

Figure 8: Examples of weak orders with different order types.

A \(c\)-hover is the following type of transformation:

\[
C_1 \succ \cdots \succ C_j \succ \{c\} \succ C_{j+2} \succ \cdots \succ C_k
\]

\[
\iff C_1 \succ \cdots \succ C_j \cup \{c\} \succ C_{j+2} \succ \cdots \succ C_k
\]

Note: \(c\) must initially lie in a singleton indifference class.

**Definition (Indifference monotonicity)**

If \(c \in f(P)\) is a winner and we apply some \(c\)-hovers, then \(c\) remains a winner.
Theorem

Approval-IRV is the only elimination scoring rule that agrees with IRV on profiles of linear orders and that is indifference monotonic.

The axioms are independent.
We can define a weak-order version of the multi-winner rule STV, giving Approval-STV. STV gives proportional representation, which has been formalized via the Proportionality for Solid Coalition property.

It has been generalized to weak orders (PJR-style).


**Definition (Generalized PSC, informal)**

If a coalition of $\alpha\%$ of voters all agree that $T \succeq C \setminus T$, then at least $\alpha\%$ of the $k$ winners should come from $T$ (or equivalently liked candidates).

**Theorem**

*Approval-STV satisfies generalized PSC for weak orders.*

Proof of some independent interest also for the linear-order variant.
Experiments

Figure 9: Average Borda score of the winner (normalized by dividing by $n$) for various datasets.

Figure 10: Frequency of agreement between the rule and linear-order IRV for various datasets.

$x$-axis: few indifferences $\rightarrow$ many indifferences
Figure 11: Map of elections, showing the difference in Borda score between the Approval-IRV and Split-IRV winner in the coin-flip model (blue: approval better than split).
### Conclusion

<table>
<thead>
<tr>
<th></th>
<th>Approval-IRV</th>
<th>Split-IRV</th>
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</thead>
<tbody>
<tr>
<td>Independence of clones</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Respecting cohesive majorities</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Indifference monotonicity</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Quality of winner</td>
<td>+</td>
<td>◯</td>
</tr>
<tr>
<td>Same winner as linear order</td>
<td>−</td>
<td>often</td>
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<tr>
<td>Generalized PSC</td>
<td>✓</td>
<td>✓</td>
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*Table 1: Comparison of properties satisfied by the rules.*