Preferences Single-Peaked on a Circle

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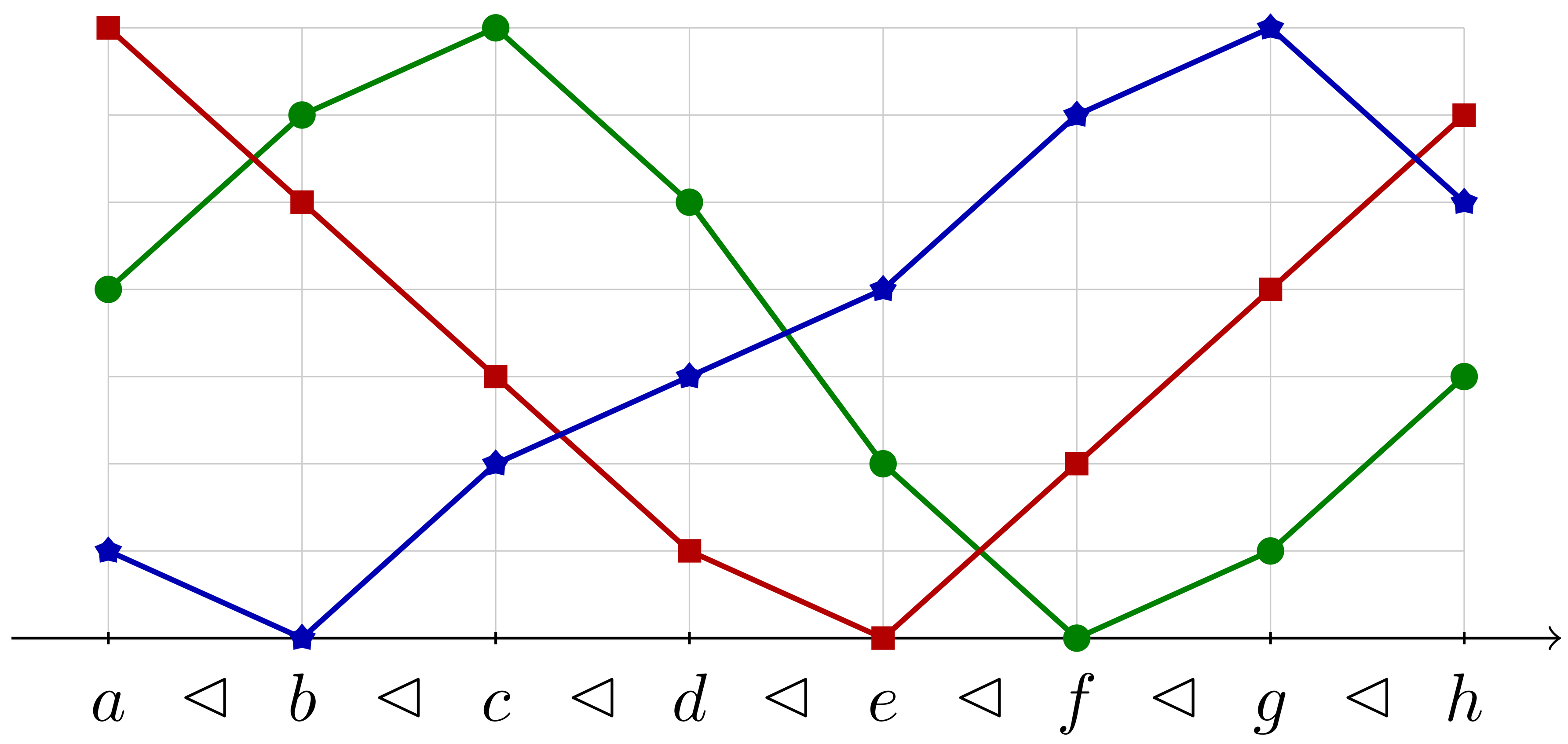
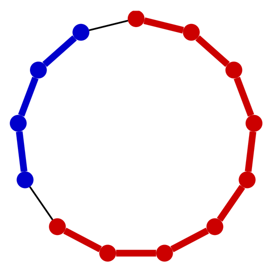
Definition

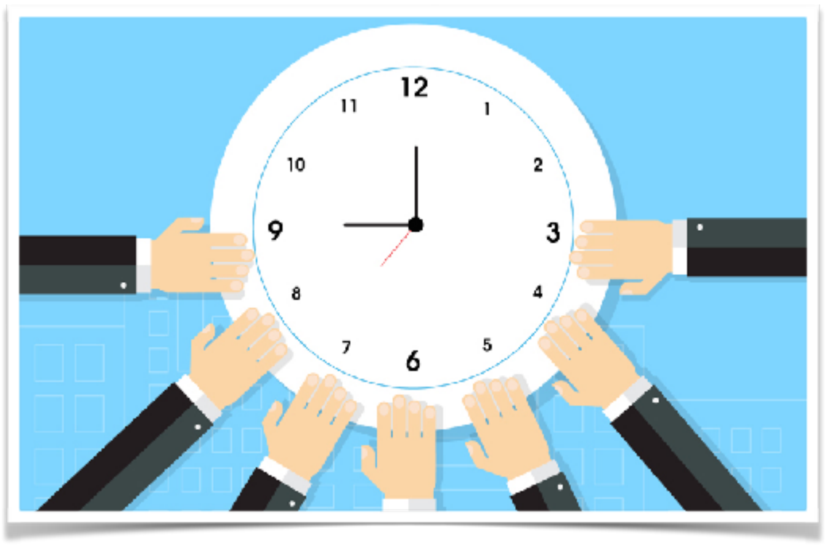


A linear preference order is single-peaked on a circle C if the circle can be cut at some point so that the preference order is single-peaked on the resulting line.

Equivalently, every top-initial segment of each vote forms an interval of the circle.



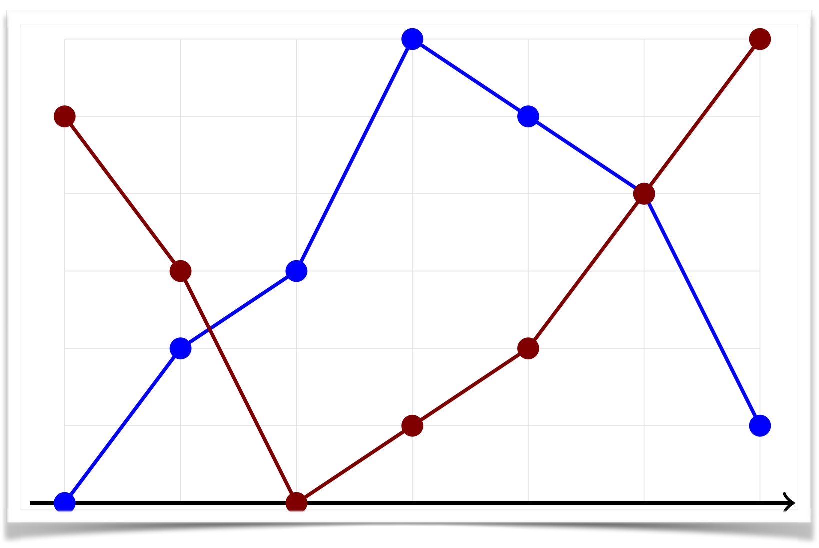
Examples and Motivation



Agreeing on a meeting time on the 24 hour clock.

Scheduling a video call across time zones.

Facility location on a circle, for example an airport on the boundary of a city.



Mix single-peaked and -caved votes on the same axis, allowing extreme opinions.

Recognition Algorithm

There exists an $O(mn)$ time algorithm that given a preference profile decides whether it is single-peaked on some circle, and if so returns a suitable circle C.

This algorithm is *certifying*: if the input profile is not SPOC, it returns one of finitely many *forbidden subprofiles*.

Young's rule

- Young's voting rule selects those alternatives that can be made a Condorcet winner by *deleting* a minimum number of voters.
- It is NP-hard to calculate in general, but *poly-time* for SPOC.
- We can efficiently calculate the Young score of any given alternative when the input profile is SPOC.

Majority Relation & Kemeny

- The Condorcet cycle *xyz, yzx, zxy* is SPOC, so SPOC profile need not admit a Condorcet winner.
- In fact, SPOC does not guarantee *anything at all* about the majority relation: McGarvey's theorem can be proven using only SPOC profiles.
- Recall that Kemeny's rank aggregation rule selects a *consensus* ranking of minimum total Kendall-tau distance to the input rankings.
- Kemeny remains NP-hard to calculate for SPOC profiles by McGarvey's theorem for SPOC.

Axiomatics & Impossibilities

- Median rule cannot be extended to SPOC.
- Gibbard-Satterthwaite can still be proven: There exists no non-imposing non-dictatorial strategy proof voting rule even on SPOC profiles.
- Moulin's no-show paradox can also be proven.

Multiwinner Rules

- Several NP-hard multiwinner voting rules become easy for profiles that are SPOC.
- This includes Chamberlin-Courant, Proportional Approval Voting (PAV), and OWA-based rules.
- The proof proceeds by encoding these rules as integer programs (ILPs) which become *totally unimodular* and thus polynomially solvable for SPOC input after some algebraic manipulation.

Comparison to Other Concepts

	SINGLE PEAKED	SINGLE PEAKED ON A TREE	SINGLE PEAKED ON A CIRCLE
AXIOMATIC PROPERTIES	⊕⊕	⊕	⊖⊖
ALGORITHMIC USEFULNESS	⊕⊕	⊖	⊕