

# Pareto-optimal Allocation of a Graph

Ayumi Igarashi

University of Kyushu

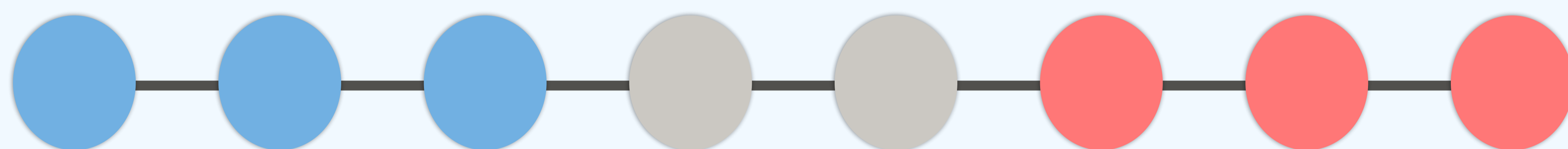
Dominik Peters

University of Oxford

**PROBLEM:** Allocate connected bundles of indivisible goods, with additive utilities.

**PO is not compatible with EF1** envy-free  
up to 1 good

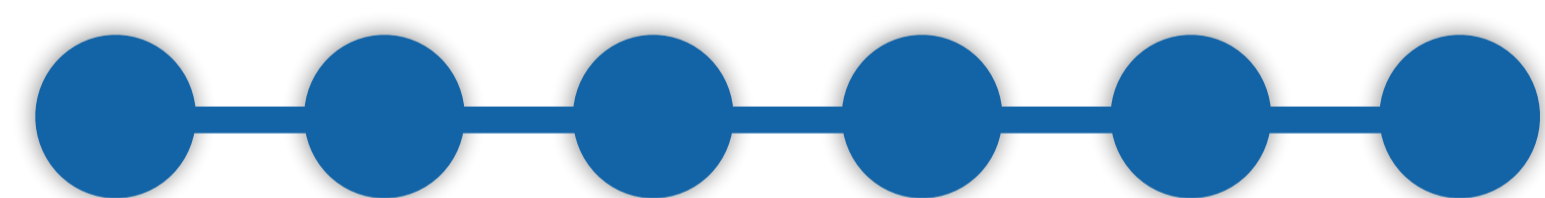
when bundles need to be connected



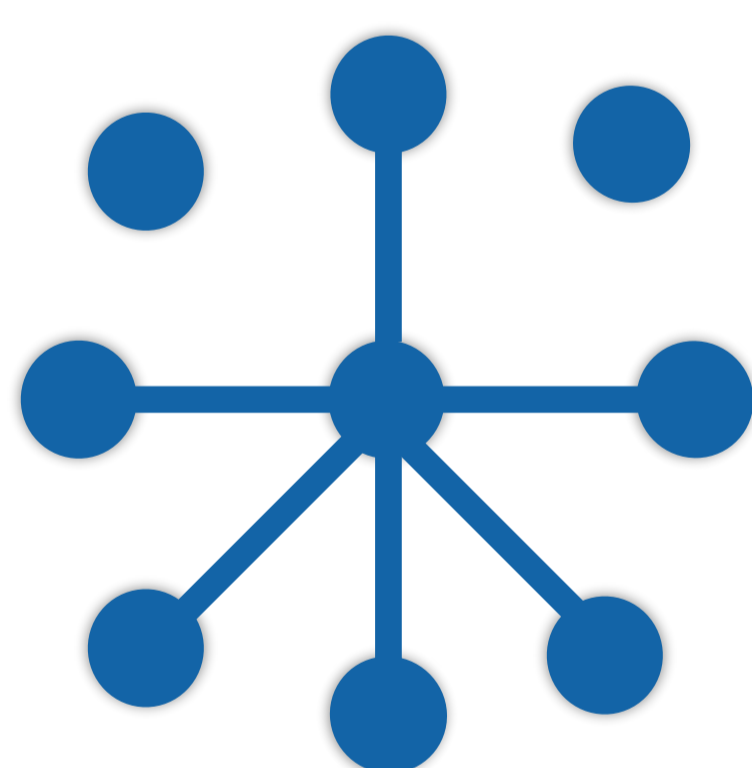
	0	0	0	1	1	0	0	0
	1	1	1	0	0	1	1	1

so Nash fails EF1

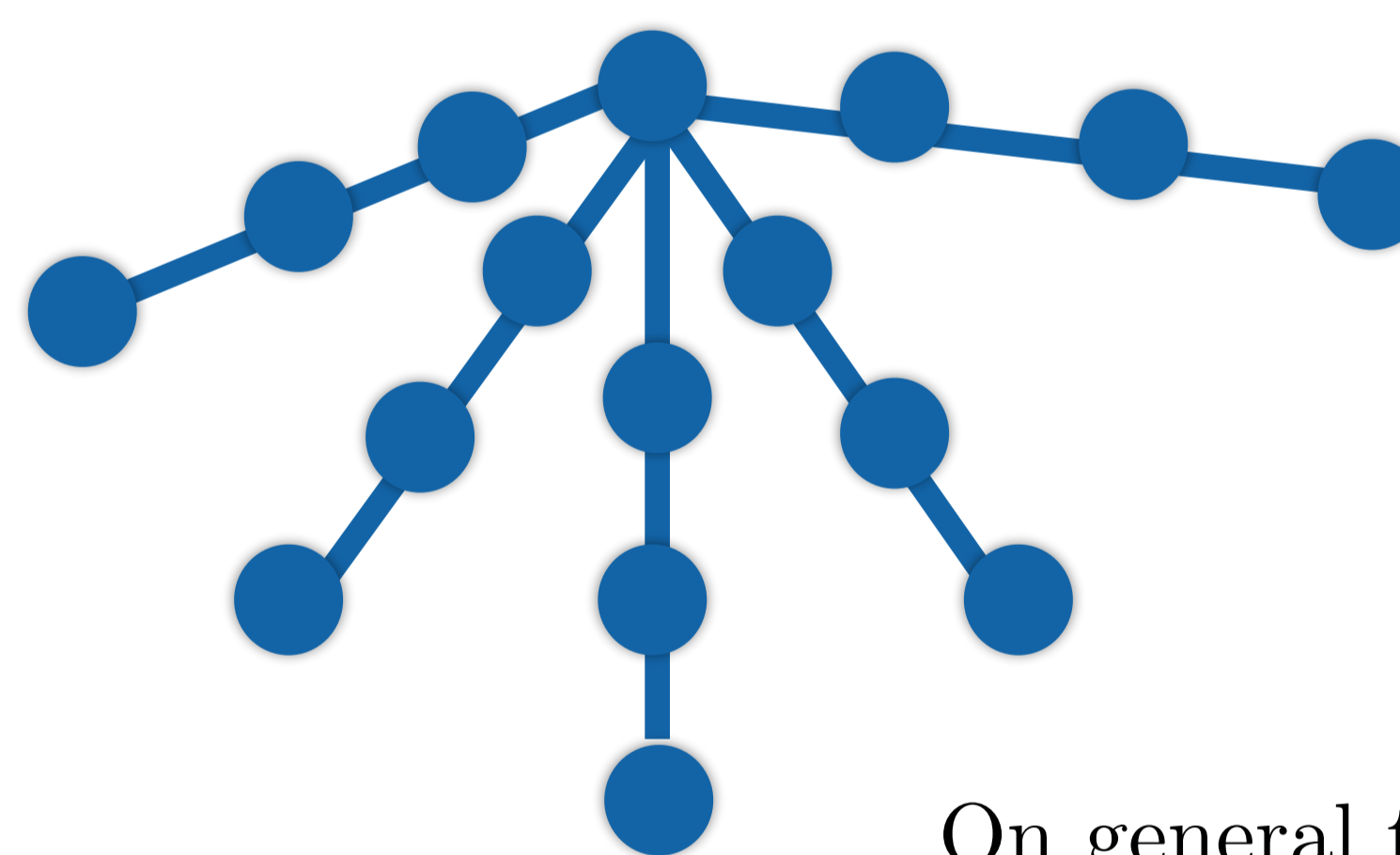
## Finding some PO allocation



On paths, version of serial dictatorship gives a PO allocation  
(but welfare max. is hard)



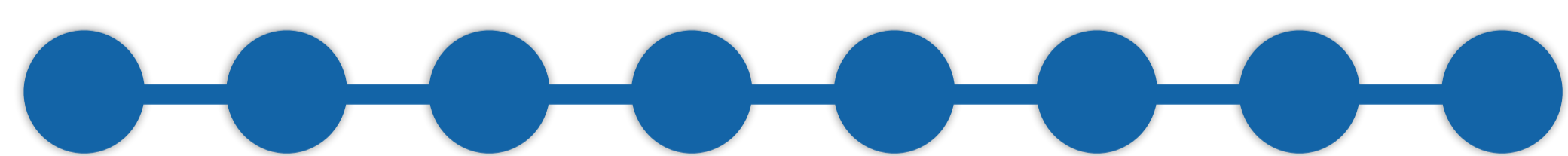
On stars, can use matching to maximise utilitarian welfare



On general trees, NP-hard (under Turing red.) to produce any PO allocation

without connectivity, this problem is trivial

## Finding some PO + MMS allocation

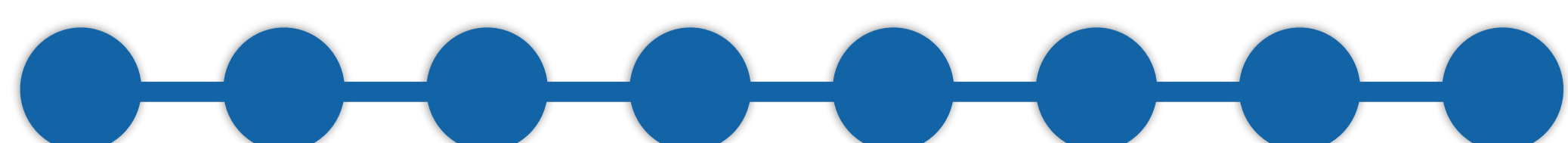


On paths, NP-hard (under Turing red.) to produce any PO + MMS allocation

even hard for PO +  $\alpha$ -MMS for any  $\alpha > 0$

MMS allocations always exists on trees, so existence of PO + MMS is guaranteed

## Deciding existence of PO + EF1



On paths, NP-hard to decide whether a PO + EF1 allocation exists