Summer School on Computational Social Choice
Computing Desirable Collective Decisions II
Distortion in Social Choice & Beyond

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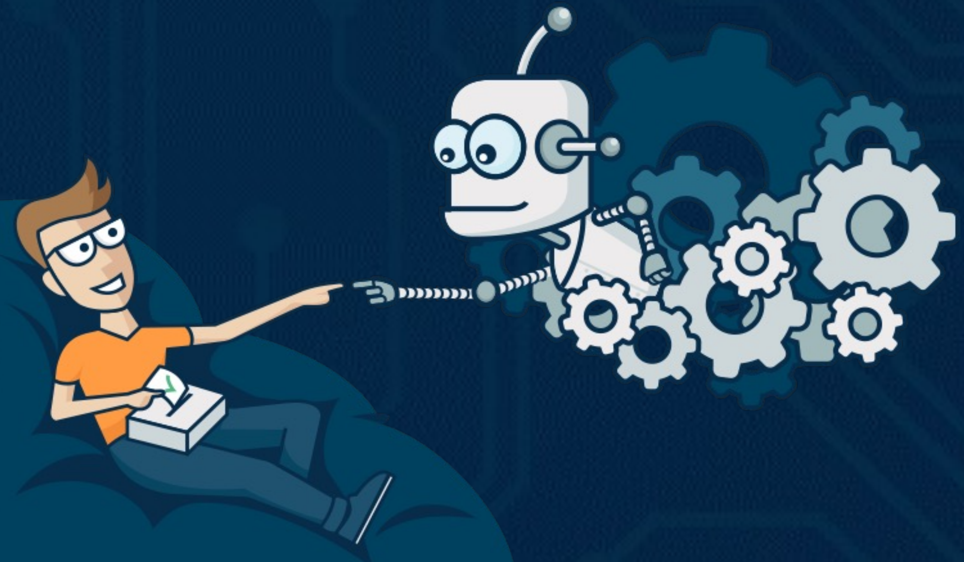


2023-07-18

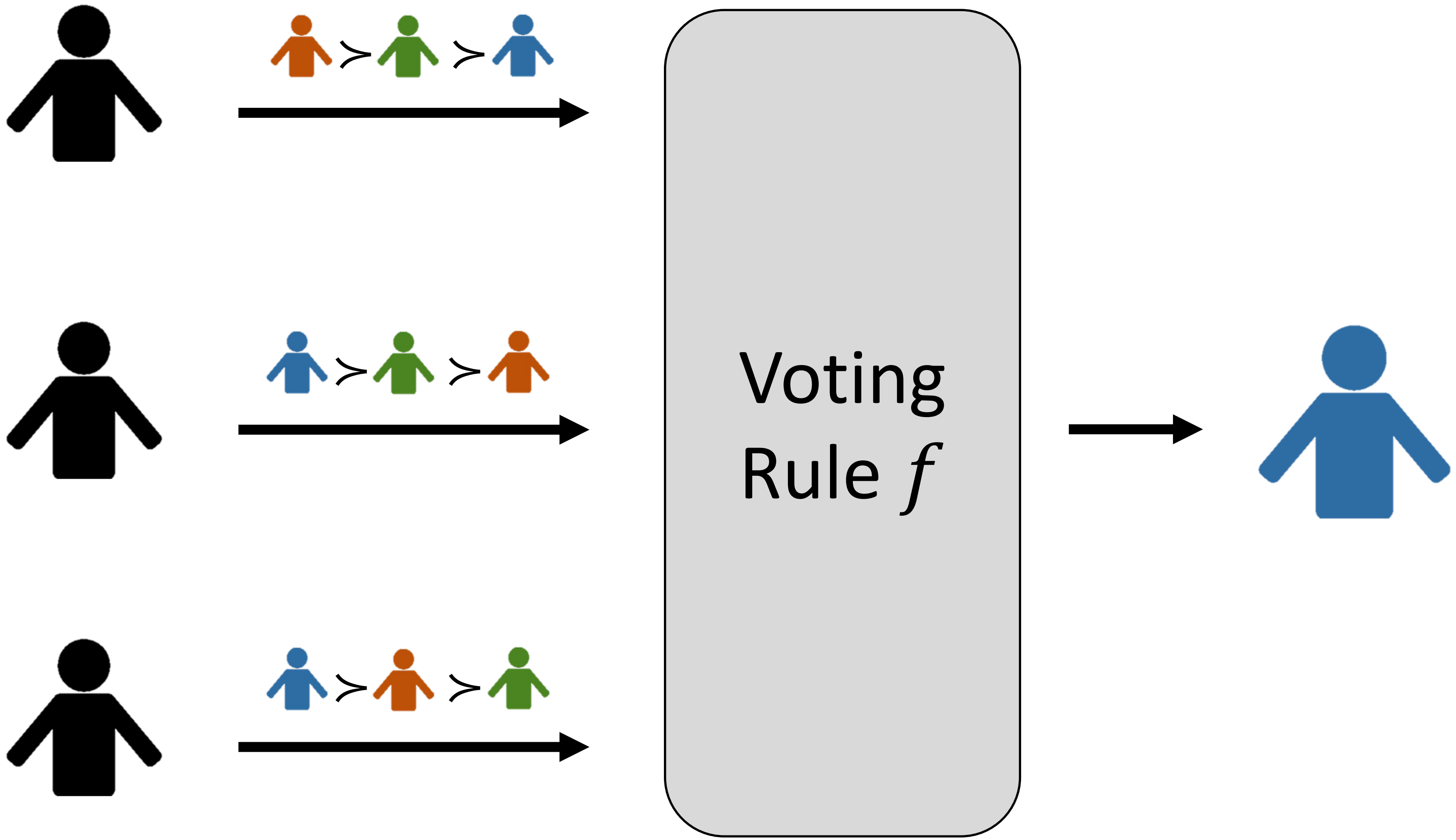
Outline

- Introduction
  - Applications of voting
  - Motivating the distortion framework
- Utilitarian distortion framework
  - Model
  - Known results
- Metric distortion framework
  - Model
  - Known results
- Applications beyond voting

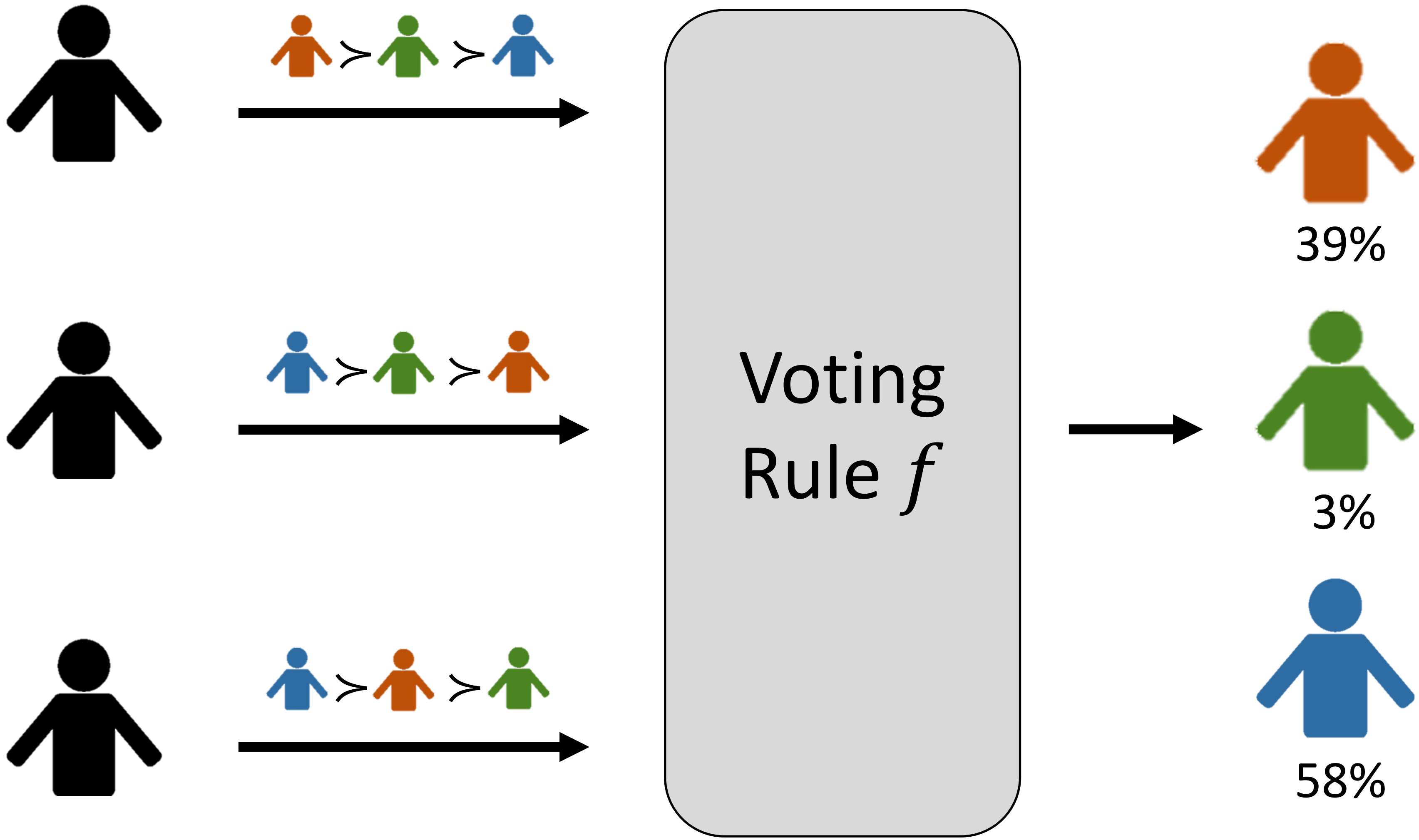
Voting

Algorithm for aggregating individual preferences to make collective decisions

Voting with Ranked Ballots



Randomized Voting with Ranked Ballots



Applications of Randomized Voting

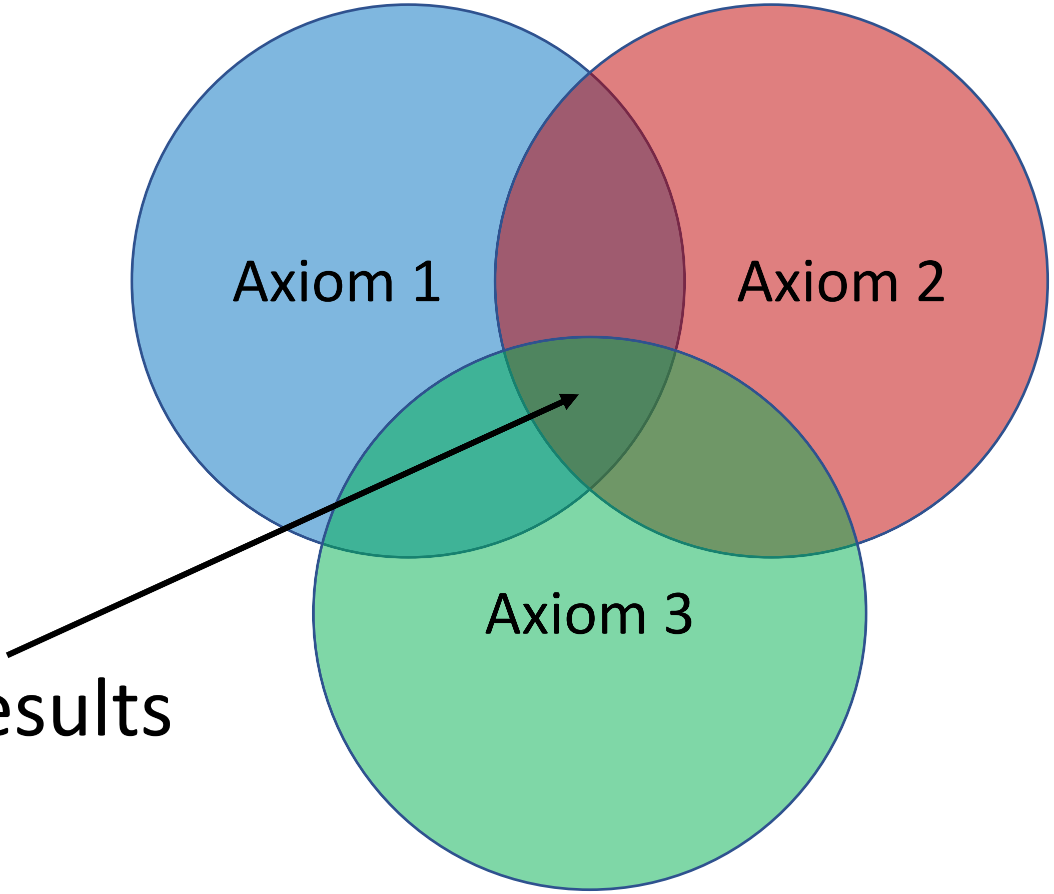
- Interpretation 1: Randomization
  - ⛔ Probably inappropriate for high-stakes political elections
  - Low stakes decisions like “which restaurant for lunch?”
  - Ensemble-leaning based recommendation engines

- Interpretation 2: Resource division
  - Foundation splitting its budget between grantees
  - Plan a workshop schedule (posters, talks, coffee, lunch, …)
  - Split a parliament between parties
  - Repeated decisions (seminar weekday, lunch restaurant)

Traditional Analysis: The Axiomatic Method

- Condorcet consistency
  - Whenever there exists an alternative a such that for every other alternative b a strict majority prefer a to b, the voting rule must select a.
- Weak monotonicity
  - If the voting rule selects alternative a in an instance and a moves up in the rankings of some of the voters, the voting rule must continue to select a.
- Axioms are qualitative
  - A voting rule either satisfies an axiom or it does not

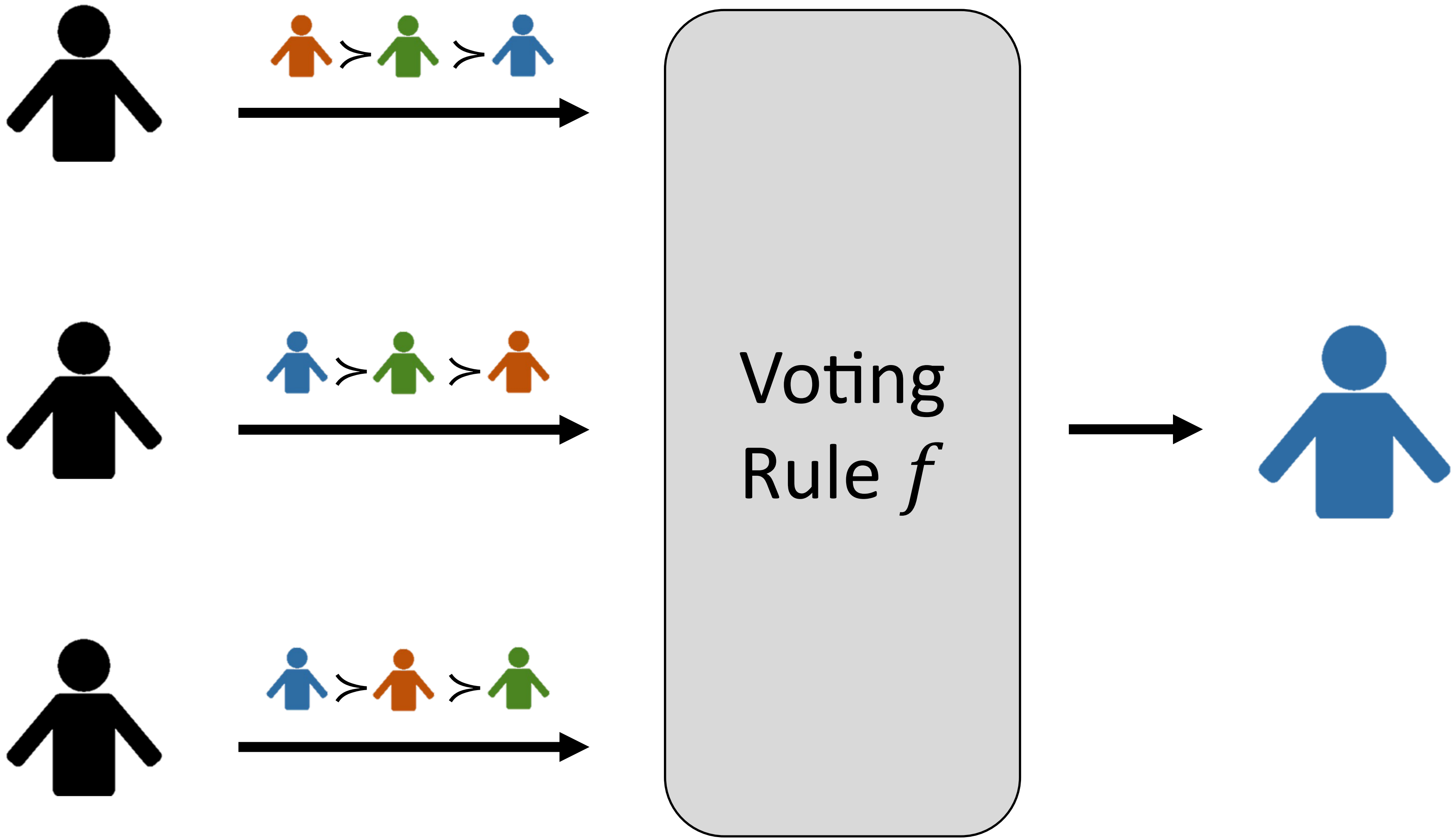
Axiomatic Method



Impossibility Results

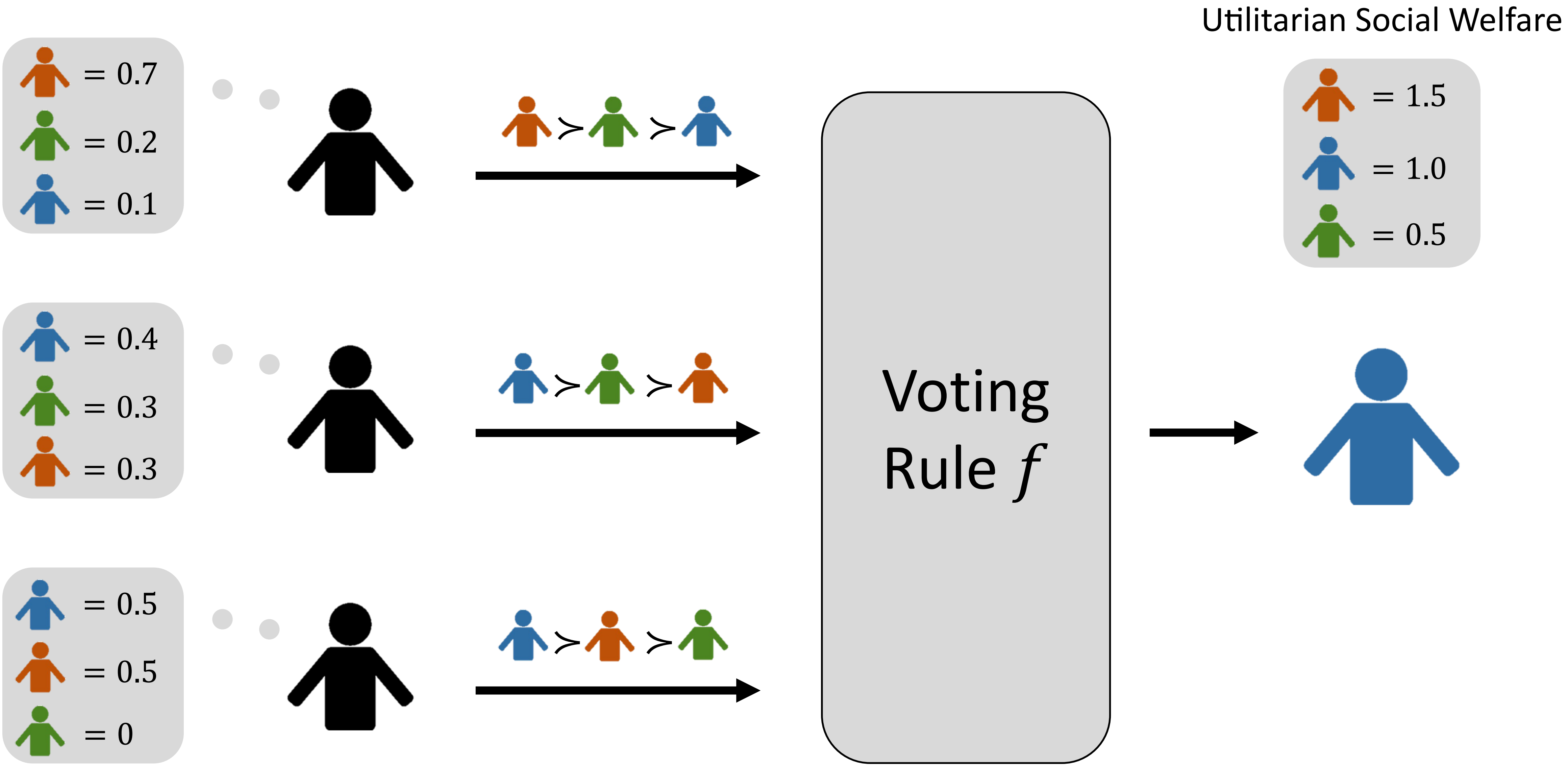
...disagreement about rules

Voting with Ranked Ballots



Utilitarian Voting

[Procaccia, Rosenschein, 2006]



No Access to Utilities

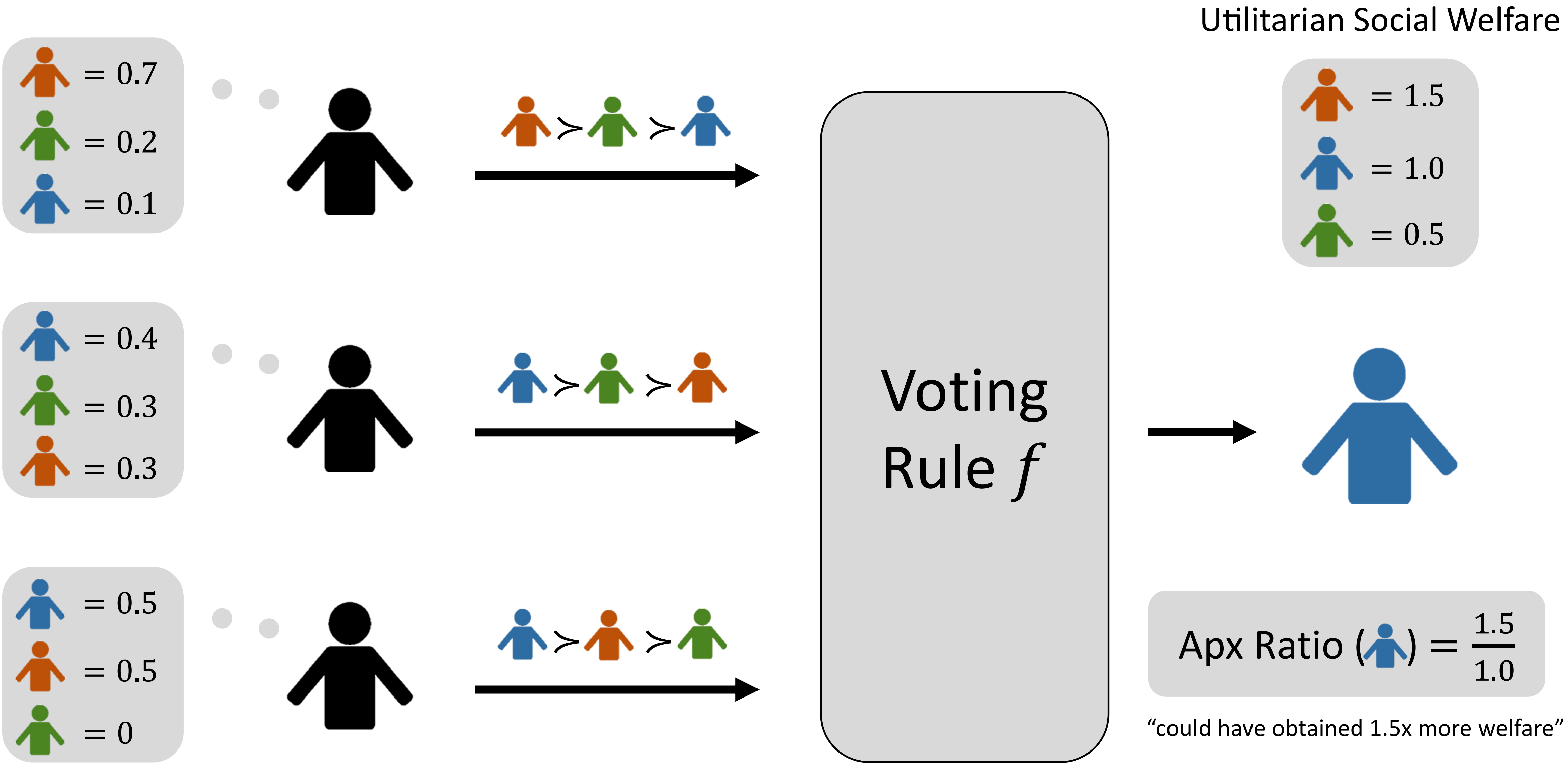
Even if voters have utilities, we may not know them, for many reasons.

- Easier elicition
  - Higher cognitive effort to assign utilities than to rank alternatives
  - It may be costly to figure out utilities (e.g. computation time to simulate consequences)
- Less communication
- Utilities are simply unknown or unknowable
- Privacy

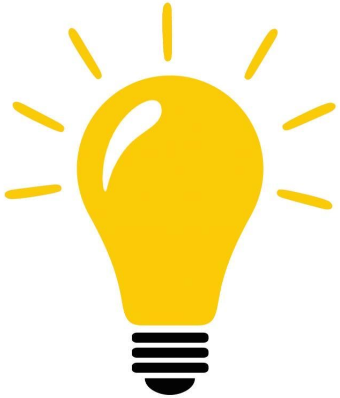
- leads to “implicit utilitarian voting”: voting rule only knows the ranking, but gets evaluated on the utilities.

Utilitarian Voting

[Procaccia, Rosenschein, 2006]



Optimal Voting Rules with Ranked Ballots



Minimize distortion
(Worst-case approximation ratio for utilitarian social welfare)

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  - Model
  - Known results
- Applications beyond voting

Voting with Ranked Ballots

- N = set of n voters
- A = set of m alternatives
  - $\Delta(A)$ = set of distributions over A
- $\vec{\succ}$ = observed ranked preference profile
  - $\succ_i$ = preference ranking of voter i
  - $a \succ_i b$ means the voter ranks a higher than b
- (Randomized) Voting rule f
  - Maps every preference profile $\vec{\succ}$ to a distribution over alternatives $f(\vec{\succ}) = x \in \Delta(A)$
  - We say that f is deterministic if $f(\vec{\succ})$ has singleton support for every $\vec{\succ}$

Utilitarian Distortion

1. There exists an underlying utility profile $\vec{u}$ such that for each $i \in N$:
   - Consistency (denoted $u_i \rhd \succ_i$): $\forall a, b : a \succ_i b \Rightarrow u_i(a) \geq u_i(b)$
   - Unit-sum: $u_i(a) \geq 0,\ \sum_a u_i(a) = 1$
     - We'll also consider unit-range: $u_i(a) \geq 0, \max_a u_i(a) = 1$
   - Linear extension to distributions: For $x \in \Delta(A)$, $u_i(x) = \sum_a u_i(a) \cdot x(a)$
2. If we knew the utilities, we would want to maximize the (utilitarian) social welfare
   - $sw(x, \vec{u}) = \sum_{i \in N} u_i(x)$ [by linearity, this optimum is attained by an alternative]
3. Because this is impossible given the limited ranked information, we want to best approximate the social welfare in the worst case.

Utilitarian Distortion

- Distortion

$$\mathrm{dist}(x, \vec{\succ}) = \sup_{\vec{u} \triangleright \vec{\succ}} \frac{\max_{a \in A} sw(a, \vec{u})}{sw(x, \vec{u})}$$

- Given voting rule f

$$dist(f) = \max_{\vec{\succ}} \mathrm{dist}\left(f(\vec{\succ}), \vec{\succ}\right)$$

What is the lowest possible $dist(f)$? Which voting rule achieves it?

Example (deterministic)

Voters

Alternatives

$1 : a \succ b \succ c$

$^1/_3 \quad ^1/_3 \quad ^1/_3$

$2 : b \succ a \succ c$

$1 \quad 0 \quad 0$

$3 : a \succ c \succ b$

$^1/_3 \quad ^1/_3 \quad ^1/_3$

- Suppose we choose a:
  - How much better can b be?

$$\max_{\vec{u} \rhd \vec{\succ}} \frac{sw(b,\vec{u})}{sw(a,\vec{u})} = \frac{^1/_3 + 1 + ^1/_3}{^1/_3 + 0 + ^1/_3} = \frac{5}{2}$$

  - How much better can c be?

$$\max_{\vec{u} \rhd \vec{\succ}} \frac{sw(c,\vec{u})}{sw(a,\vec{u})} = \frac{^1/_3 + 0 + ^1/_3}{^1/_3 + 0 + ^1/_3} = 1$$

  - Hence, $dist(a, \vec{\succ}) = \frac{5}{2} = 2.5$
  - Similarly, compute $dist(b, \vec{\succ}) = 7$ and $dist(c, \vec{\succ}) = \infty$
    - a has lower distortion than b and c

Example (randomized)

Voters

Alternatives

$1 : a \succ b \succ c$

$1 \quad 0 \quad 0$

$2 : b \succ a \succ c$

${}^{1}/_{3} \quad {}^{1}/_{3} \quad {}^{1}/_{3}$

$3 : a \succ c \succ b$

$1 \quad 0 \quad 0$

- Among deterministic choices, a is best with distortion 2.5
- With randomization, we can achieve lower distortion.
- On this profile, $x = (a: 0.5882, b: 0.4118, c: 0)$ has distortion 1.54 (best possible).

Utilitarian Distortion

- Instance-optimal rules
  - Deterministic f^*_{det}: Maps every preference profile $\vec{\succ}$ to $a^* \in \arg\min_{a \in A} \text{dist}(a, \vec{\succ})$
  - Randomized f^*_{rand}: Maps every preference profile $\vec{\succ}$ to $x^* \in \arg\min_{x \in \Delta(A)} \text{dist}(x, \vec{\succ})$
  - Have the lowest distortion on each $\vec{\succ}$, and therefore in the worst case over all $\vec{\succ}$

Are the instance-optimal rules polytime computable?
Do they have a nice analytical structure?

Optimal Deterministic Distortion

- Theorem [Caragiannis, Procaccia, 2011; Caragiannis, Nath, Procaccia, Shah, 2017]
  - For deterministic aggregation of ranked ballots, the optimal distortion is $\Theta(m^2)$ and the instance-optimal rule f_{det}^* is polytime computable.

- Proof (lower bound):
  - High-level approach:
    - Take an arbitrary voting rule f
    - Construct a preference profile $\vec{\succ}$
    - Let f choose a winner a on $\vec{\succ}$
    - Reveal a bad utility profile $\vec{u}$ consistent with $\vec{\succ}$ in which a is $\Omega(m^2)$ factor worse than the optimal alternative

Deterministic Rules

- Proof (lower bound):
  - Let f be any deterministic voting rule
  - Consider $\vec{\succ}$ on the right
  - Case 1: $f(\vec{\succ}) = a_m$
    - Infinite distortion. Why?
  - Case 2: $f(\vec{\succ}) = a_i$ for some $i < m$
    - Bad utility profile $\vec{u}$ consistent with $\vec{\succ}$
      - Voters in column i have utility $1/m$ for every alternative
      - All other voters have utility $1/2$ for their top two alternatives
    - $\text{sw}(a_i, \vec{u}) = \frac{n}{m-1} \cdot \frac{1}{m}$, $\text{sw}(a_m, \vec{u}) \geq \frac{n - n/(m-1)}{2} = \Omega(n)$
    - Distortion = $\Omega(m^2)$

$n/(m-1)$ voters per column			
a_1	a_2	…	a_{m-1}
a_m	a_m	…	a_m
⋮	⋮	⋮	⋮

Deterministic Rules

- Proof (upper bound):
  - Plurality rule: Select an alternative a that is the top choice of the most voters
  - For this plurality winner:
    - At least $^n/_m$ voters have a as their top choice (pigeonhole principle)
    - Every voter has utility at least $^1/_m$ for their top choice (pigeonhole principle)
  - Hence, for every consistent utility profile $\vec{u}$:
    - $sw(a, \vec{u}) \geq {}^n/_{m^2}$
    - $sw(a^*, \vec{u}) \leq n$ for every alternative a^*
  - $dist(a, \vec{\succ}) = O(m^2)$

Optimal Randomized Distortion

- Theorem [Boutilier, Caragiannis, Haber, Lu, Procaccia, and Sheffet, 2015]
  - For randomized aggregation of ranked ballots:
    - There is a voting rule with distortion $O(\sqrt{m} \cdot \log^* m)$.
    - Every voting rule has distortion at least $\Omega(\sqrt{m})$.
    - The instance-optimal rule f^*_{rand} is computable in polynomial time.

- Proof (lower bound):
  - Same high-level approach:
    - Take an arbitrary *randomized* voting rule f
    - Construct a preference profile $\vec{\succ}$
    - Let f choose a distribution x over alternatives
    - Reveal a bad utility profile $\vec{u}$ consistent with $\vec{\succ}$ in which the expected social welfare under x is $\Omega(\sqrt{m})$ factor worse than the optimal social welfare

Randomized Rules

- Proof (lower bound):
  - Let f be an arbitrary rule
  - Consider $\vec{\succ}$ on the right with $\sqrt{m}$ special alternatives
  - f returns distribution x in which at least one special alternative (say a_1) must be chosen with prob. at most $1/\sqrt{m}$
  - Bad utility profile $\vec{u}$ consistent with $\vec{\succ}$:
    - All voters ranking a_1 first have utility 1 for a_1
    - All other voters have utility $1/m$ for every alternative
    - $\text{sw}(a_1, \vec{u}) = \Theta\left(n/\sqrt{m}\right)$ but $sw(a, \vec{u}) \le n/m$ for every other alternative a
    - $sw(x, \vec{u}) \le \left(1/\sqrt{m}\right) \cdot \Theta\left(n/\sqrt{m}\right) + \left(1 - 1/\sqrt{m}\right) \cdot (n/m) = O(n/m)$
    - Hence, $dist(x, \vec{u}) = \Omega(\sqrt{m})$

$n/\sqrt{m}$ voters per column			
a_1	a_2	…	$a_{\sqrt{m}}$
⋮	⋮	⋮	⋮

Optimal Randomized Distortion

- Harmonic Rule
  - The rule that achieves $O(\sqrt{m} \cdot \log^* m)$ distortion is complicated and artificial (it only makes sense if you want low distortion) and is unlikely to generalize
  - [Boutilier et al. 2015] propose a simpler rule that achieves $O\left(\sqrt{m \cdot \log m}\right)$ distortion

Harmonic Rule

- Each voter i awards $^1/_r$ points to her r^{th} ranked alternative for every $r \in \{1, \ldots m\}$
- Harmonic score of alternative a, denoted $hsc(a, \vec{\succ})$, is the total point awarded to a
- With probability ½, choose each $a \in A$ with probability proportional to $hsc(a, \vec{\succ})$
- With probability ½, choose each $a \in A$ uniformly at random

- Key proof idea:
  - $hsc(a, \vec{\succ}) \geq sw(a, \vec{u})$ for every a, while $\sum_a hsc(a, \vec{\succ}) = O(\log m) \cdot \sum_a sw(a, \vec{u})$

Optimal Randomized Distortion

- Theorem [Ebadian, Kahng, Peters, Shah, 2022]
  - For randomized aggregation of ranked ballots, the optimal distortion is $\Theta(\sqrt{m})$.

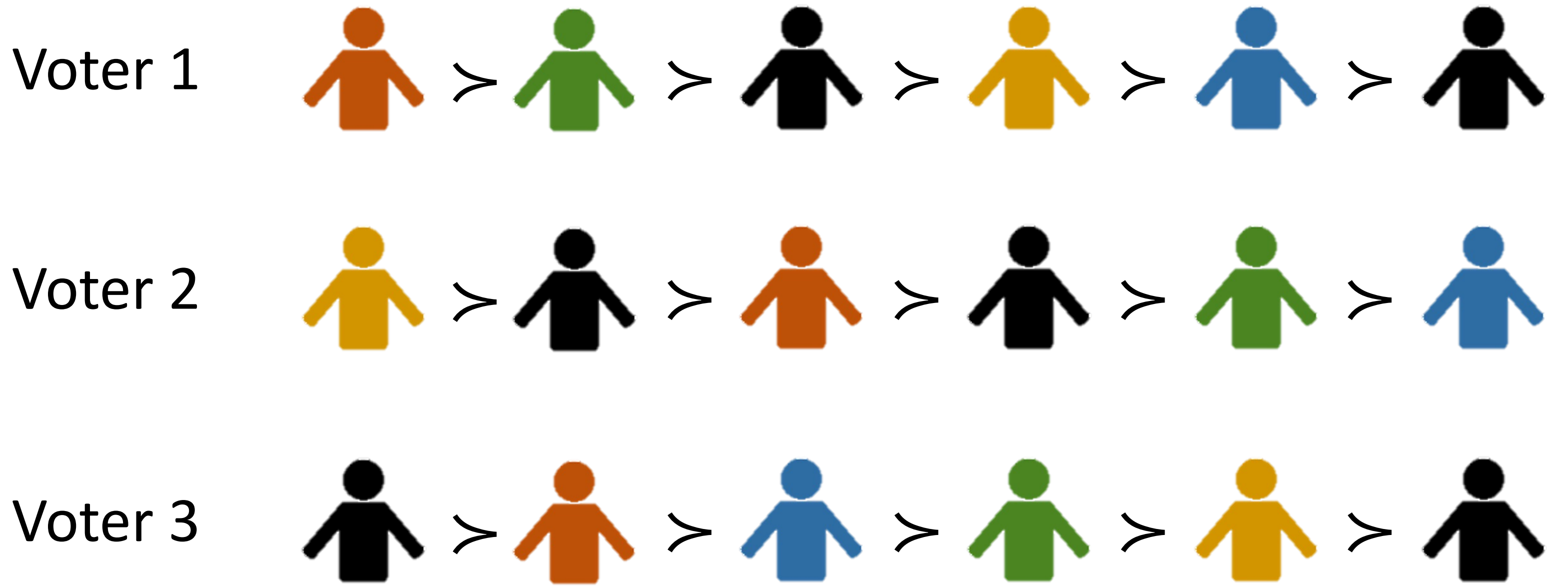
- Proof via three steps:

I. Define "stable lotteries"

II. Prove the existence (and efficient computation) of stable lotteries via the minimax theorem

III. Derive $O(\sqrt{m})$ distortion using stable lotteries

Step I: Define Stable Lotteries



- For a set of alternatives $S = \{$$\}$ and an alternative $a =$ 

$$V(a, S) = |\{i \in N : a \succ_i b, \forall b \in S\}| = 2$$

- A set of size k is stable if $V(a, S) \leq {}^{n}/_{k}$ for every $a \in A$
- Lottery $\mathcal{S}$ over sets of size k is stable if $\mathbb{E}_{S \sim \mathcal{S}}[V(a, S)] \leq {}^{n}/_{k}$ for every $a \in A$

$k = 1$:
maximal lottery

Step II: Prove Stable Lotteries Exist

- Theorem: For every k, a stable lottery over committees of size k exists.
- Proof (skip):
  - $\min_{\mathcal{S}} \max_{a \in A} \mathbb{E}_{S \sim \mathcal{S}}[V(a,S)] \leq \min_{\mathcal{S}} \max_{x \in \Delta(A)} \mathbb{E}_{S \sim \mathcal{S}, a \sim x}[V(a,S)]$
    $= \max_{x \in \Delta(A)} \min_{\mathcal{S}} \mathbb{E}_{S \sim \mathcal{S}, a \sim x}[V(a,S)] \leq \frac{n}{k}$
  - For any $x \in \Delta(A)$, consider the lottery $\mathcal{S}^*$, where we sample k alternatives i.i.d. according to x and replace any duplicates with arbitrary other alternatives
  - For each voter i:

$$\Pr_{S \sim \mathcal{S}^*, a \sim x}[a \succ_i b, \forall b \in S] \leq \frac{1}{k+1}$$

  - Hence:

$$\mathbb{E}_{S \sim \mathcal{S}^*, a \sim x}[V(a,S)] \leq \frac{n}{k+1} < \frac{n}{k} \quad \blacksquare$$

Step III: Proof of $O(\sqrt{m})$ Distortion

Stable Lottery Rule

- With probability ½ , find a stable lottery $\mathcal{S}$ over sets of size $\sqrt{m}$, sample $S \sim \mathcal{S}$, choose $a \in S$ uniformly at random
- With probability ½, choose $a \in A$ uniformly at random

- Theorem: Stable lottery rule achieves $O(\sqrt{m})$ distortion.
  - Let a^* be an alternative maximizing social welfare
  - For any S: $sw(a^*, \vec{u}) \leq V(a^*, S) + \sum_{b \in S} sw(b, \vec{u})$
  - Taking expectation over $S \sim \mathcal{S}$:

$$
\begin{aligned}
sw(a^*, \vec{u}) &\leq \mathbb{E}_{S \sim \mathcal{S}}[V(a^*, S)] + \mathbb{E}_{S \sim \mathcal{S}}[\textstyle\sum_{b \in S} sw(b, \vec{u})] \\
&\leq 2\sqrt{m} \cdot \left(\frac{1}{2} \cdot \frac{n}{m} + \frac{1}{2} \cdot \mathbb{E}_{S \sim \mathcal{S}}\left[\frac{1}{|S|} \cdot \textstyle\sum_{b \in S} sw(b, \vec{u})\right]\right) \\
&= 2\sqrt{m} \cdot sw(f(\vec{\succ}), \vec{u}) \ \blacksquare
\end{aligned}
$$

Notes

- Stable lotteries
  - Introduced by [Cheng, Jiang, Munagala, Wang, 2020], who show the existence of a stronger form of stable lotteries which bounds $V(S', S)$ for all $S' \subseteq A$
  - Requires a much more intricate proof
- Stable committees
  - 16-stable committees exist [Jiang, Munagala, Wang, 2020]: $V(a, S) \leq 16 \cdot \frac{n}{k}$ for all $a \in A$
  - Factor 16 cannot be improved to any lower than 2
  - Open question: Do 2-approximately stable committees exist?
- Lower bound
  - The lower bound from before is $\frac{\sqrt{m}}{2}$
  - Open question: A gap of factor 4 between this lower bound and the $2\sqrt{m}$ upper bound by stable lottery rule

Extensions

- Other utility classes and objective functions
- Incentives
- Ballot formats other than ranked ballots
- Committee selection
- Optimal ballot design
- Participatory budgeting
- Social welfare functions

Other Objective Functions

- Nash social welfare
  - $sw(x, \vec{u}) = \sum_{i \in N} u_i(x)$
  - $nsw(x, \vec{u}) = \left(\prod_{i \in N} u_i(x)\right)^{1/n}$
  - Provides fairness properties (proportional representation)
  - Nash social welfare is independent of individual scales
    - Any distortion upper bound with respect to unit-sum utilities holds for arbitrary utilities
- Theorem [Ebadian, Kahng, Peters, Shah, 2022]:
  - With respect to the Nash social welfare:
    - The distortion of harmonic rule is $\Theta\left(\sqrt{m \cdot \log m}\right)$
    - The distortion of stable committee rule (similar to stable lottery rule) is $\Theta(\sqrt{m})$
    - There is a randomized rule with distortion $O(\log m)$

Other Utility Classes

- Unit range utilities:
  - $u_i(a) \in [0,1]$ for all $a \in A$, $\max_a u_i(a) = 1$, $\min_a u_i(a) = 0$

- Theorem [Ebadian, Kahng, Peters, Shah, 2022]:
  - With respect to unit range utilities:
    - The distortion of harmonic rule increases to $O\left(m^{2/3} \cdot \log^{1/3} m\right)$
    - The distortion of stable lottery rule remains $O(\sqrt{m})$
    - Every randomized rule has distortion $\Omega(\sqrt{m})$

Incentives

- Strategyproofness
  - A randomized rule is strategyproof if a voter cannot increase her expected utility by misreporting her preference ranking in any instance.
- Theorem [Bhaskar, Dani, Ghosh, 2018]:
  - With respect to unit-sum utilities, the best distortion subject to strategyproofness is $\Theta\left(\sqrt{m \cdot \log m}\right)$.
  - Upper bound is achieved by harmonic rule, which is strategyproof.
- Theorem [Filos-Ratsikas, Bro Miltersen, 2014; Lee 2019]:
  - With respect to unit-range utilities, the best distortion subject to strategyproofness is $\Theta\left(m^{2/3}\right)$.
  - Note: This explains why the distortion of harmonic rule, which is strategyproof, increases to $\tilde{O}\left(m^{2/3}\right)$ for unit-range utilities
    - Harmonic rule achieves near-optimal distortion subject to strategyproofness with respect to both unit-sum and unit-range utilities!

Other Ballot Formats

- Ranked ballots + additional queries (more information than ranked ballots)
  - Value query: What is $u_i(a)$?
  - Comparison query: Is $u_i(a) \geq \alpha \cdot u_i(b)$?
  - We measure the number of queries *per voter*

- Theorem [Amanatidis, Birmpas, Filos-Ratsikas, Voudouris, 2021]:
  - For any k, it is possible to achieve distortion $O\left(\sqrt[k+1]{m}\right)$ with $O(k \cdot \log m)$ value queries
  - It is possible to achieve $O(1)$ distortion using $O(\log^2 m)$ comparison queries
  - The best distortion with λ value queries is $\Omega\left(\frac{1}{\lambda+1} \cdot m^{\frac{1}{2(\lambda+1)}}\right)$
  - …

- Many open questions:
  - E.g., $O(1)$ distortion with $O(\log m)$ value queries?

Many, Many Open Questions

- Combining extensions
  - Strategyproofness +
    - Nash welfare distortion, additive distortion, other ballots, committee selection, …
  - Committee selection or participatory budgeting +
    - Nash welfare distortion, additive distortion, …
  - Unit-range utilities +
    - Additive distortion, other ballots, committee selection, participatory budgeting, …
  - Social welfare functions?
  - …

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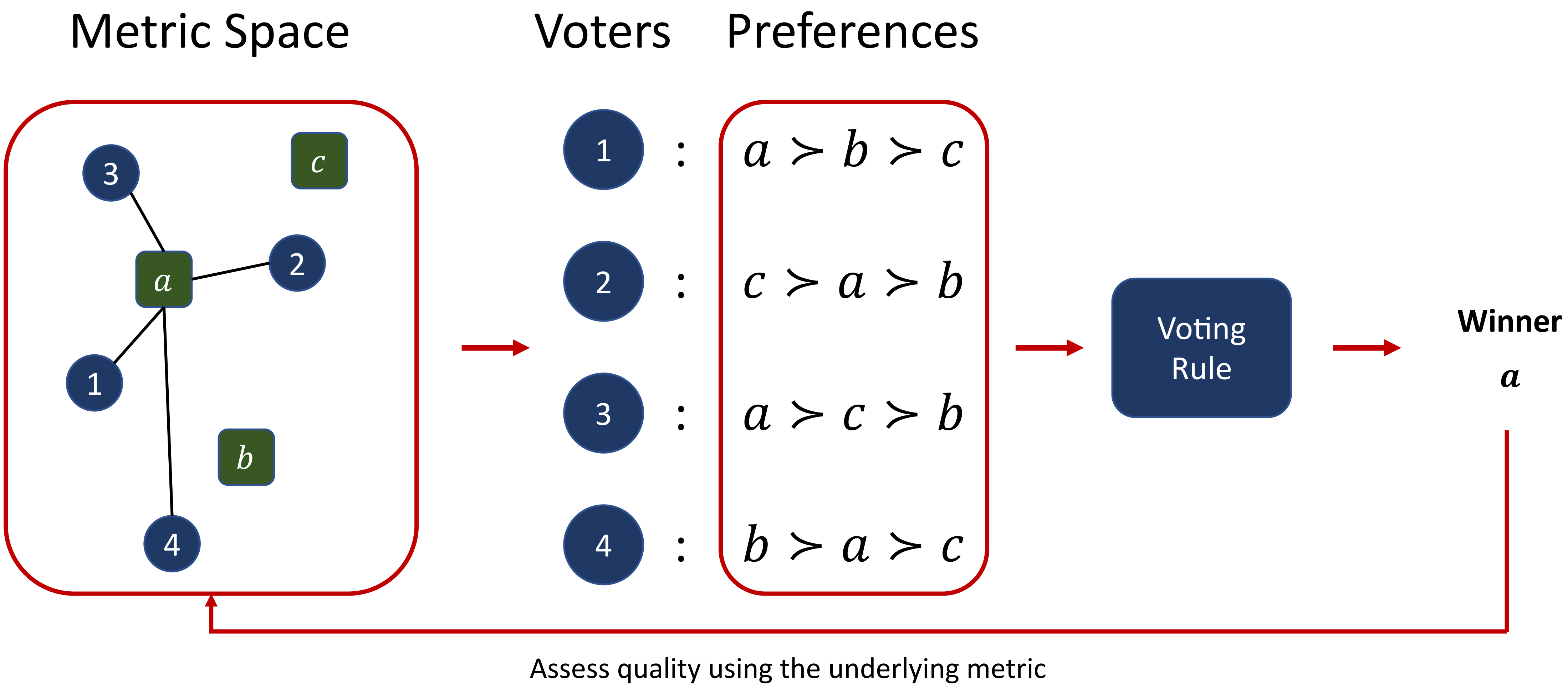
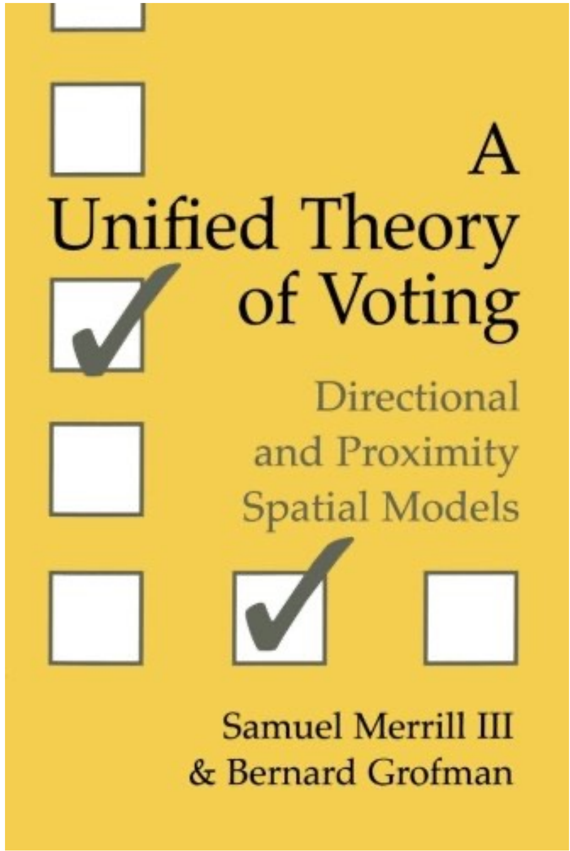
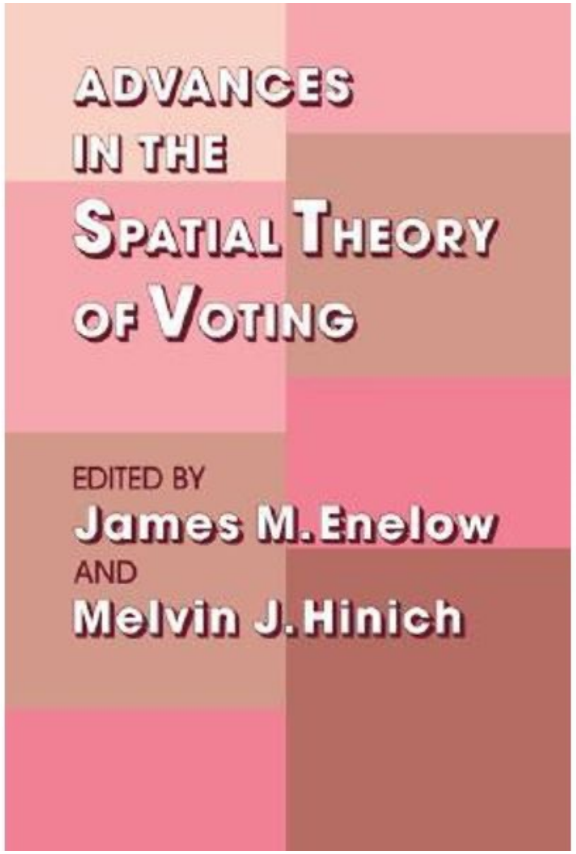
Metric Distortion

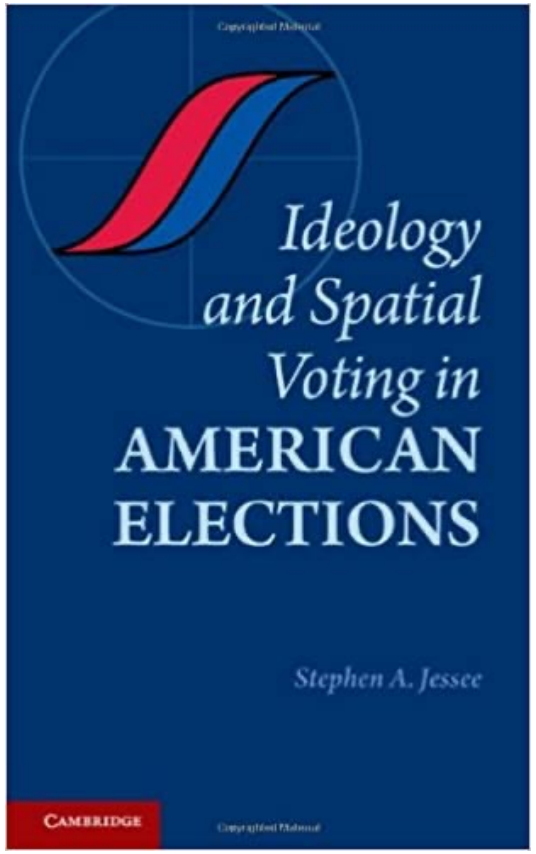
[Anshelevich, Bhardwaj, Elkind, Postl, Skowron, 2018]



Why The Metric?







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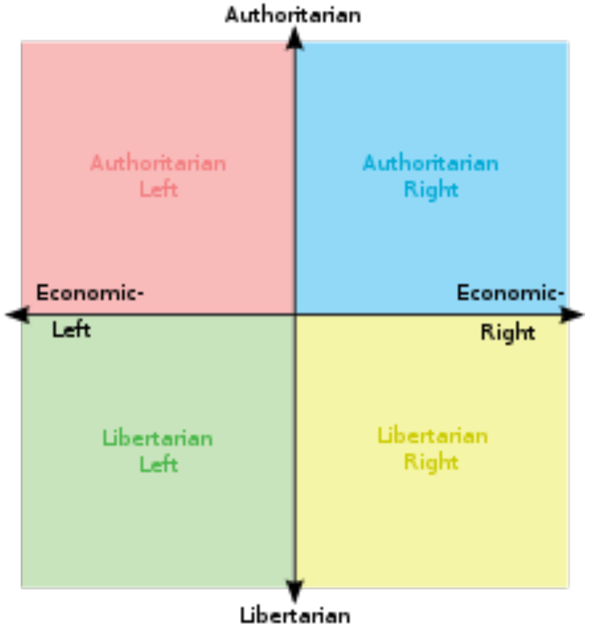
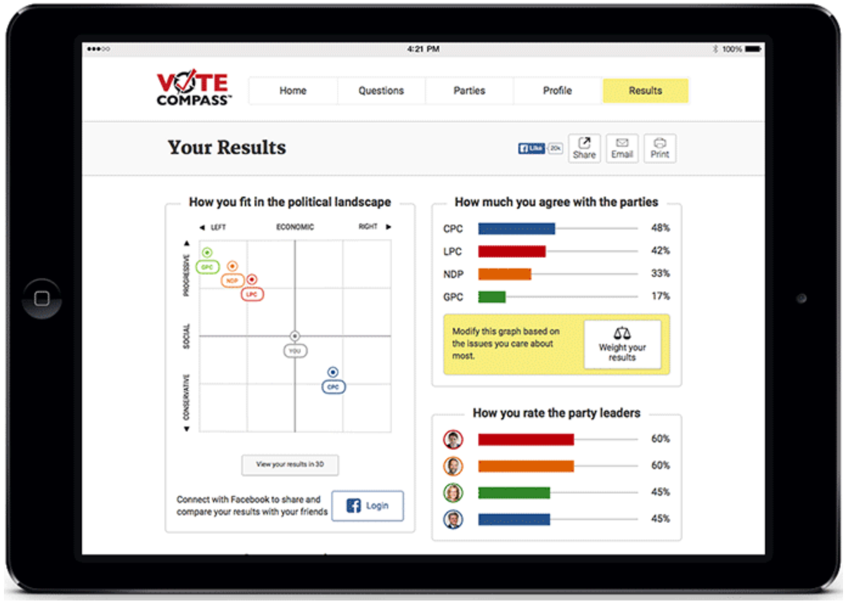


2D Models

3D Models



Popular Tools

Metric Distortion

1. There exists an underlying metric d over voters and alternatives such that:
   - Consistency (denoted $d \rhd \vec{\succ}$) : $\forall a, b : a \succ_i b \Rightarrow d(i,a) \leq d(i,b)$
   - Triangle inequality: $\forall x, y, z, d(x,y) + d(y,z) \geq d(x,z)$
   - Linear extension to distributions: For $x \in \Delta(A)$, $c_i(x) = d(i,x) = \sum_a d(i,a) \cdot x(a)$

2. If we knew the costs, we would minimize the social cost
   - $sc(x,d) = \sum_{i \in N} d(i,x)$

3. Because this is impossible given the limited ranked information, we want to best approximate the social cost in the worst case.

Metric Distortion

- Distortion

$$\mathrm{dist}(x, \vec{\succ}) = \sup_{d \rhd \vec{\succ}} \frac{sc(x,d)}{\min_{a \in A} sc(a,d)}$$

- Given voting rule f

$$dist(f) = \max_{\vec{\succ}} \mathrm{dist}\big(f(\vec{\succ}), \vec{\succ}\big)$$

What is the lowest possible distortion of deterministic and randomized rules? Which voting rules achieves it?

Lower Bound

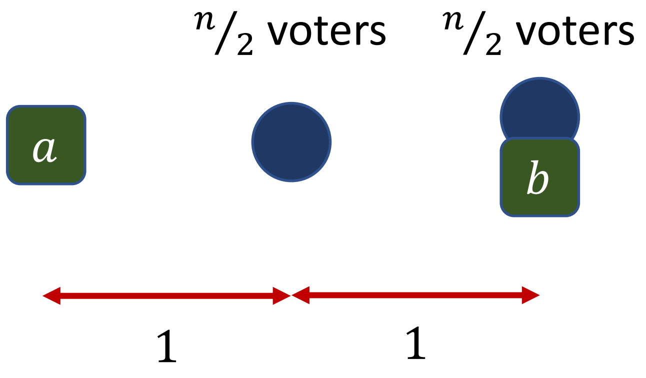
[Anshelevich, Bhardwaj, Elkind, Postl, Skowron, 2018]

- A simple lower bound of 3 (deterministic rules) with just two candidates

$^n/_2$ voters : $a \succ b$

$^n/_2$ voters : $b \succ a$

Rule selects → (w.l.o.g.) **Winner** $\boldsymbol{a}$ → Bad instance



$$sc(a,d) = 1\frac{n}{2} + 2\frac{n}{2} = 3\frac{n}{2}$$

$$sc(b,d) = 1\frac{n}{2} + 0\frac{n}{2} = \frac{n}{2}$$

$$\text{distortion} \geq \frac{sc(a,d)}{sc(b,d)} \geq 3$$

Can a deterministic rule achieve distortion 3?

Deterministic Rules

- Theorem [Anshelevich, Bhardwaj, Elkind, Postl, Skowron, 2018]:

Rule	Distortion
k-approval ($k > 2$)	Unbounded
Plurality, Borda count	$\Theta(m)$
Harmonic rule*	$O\left(\frac{m}{\sqrt{\log m}}\right), \Omega\left(\frac{m}{\log m}\right)$
Best positional scoring rule	$\Omega(\sqrt{\log m})$
Instant runoff voting (STV)	$O(\log m),\ \Omega(\sqrt{\log m})$
Copeland's rule	5
Best deterministic rule	≥ 3

*Deterministic version of the harmonic rule,
which simply picks an alternative with the largest harmonic score

- The instance-optimal deterministic rule can be computed in polynomial time by solving a number of linear programs.

- Open question: What is the best distortion achievable by any positional scoring rule?

Copeland's Rule

- Lemma [Kempe 2020b]:
  - If $(a_1, a_2, \ldots, a_\ell)$ is a sequence of alternatives such that a (weak) majority of voters prefer a_i to a_{i+1} for each $i = 1, \ldots, \ell - 1$, then $sc(a_1, d) \leq (2\ell - 1) \cdot sc(a_\ell, d)$ for every metric d consistent with the preference profile.
- Corollary:
  - It is known that Copeland's winner is in the uncovered set:
    - If a_1 is Copeland's winner, then for every other alternative a, either sequence (a_1, a) or (a_1, a_2, a) for some a_2 satisfies the condition above.
  - This explains distortion 5 of Copeland's rule
  - Lemma quite powerful, later used by [Anagnostides, Fotakis, Patsilinakos, 2021]
- Copeland's rule is Condorcet consistent
  - [Anshelevich, Bhardwaj, Elkind, Postl, Skowron, 2018]: Any voting rule can be made Condorcet consistent without losing distortion because the Condorcet winner is always a 3-approximation

Deterministic Rules

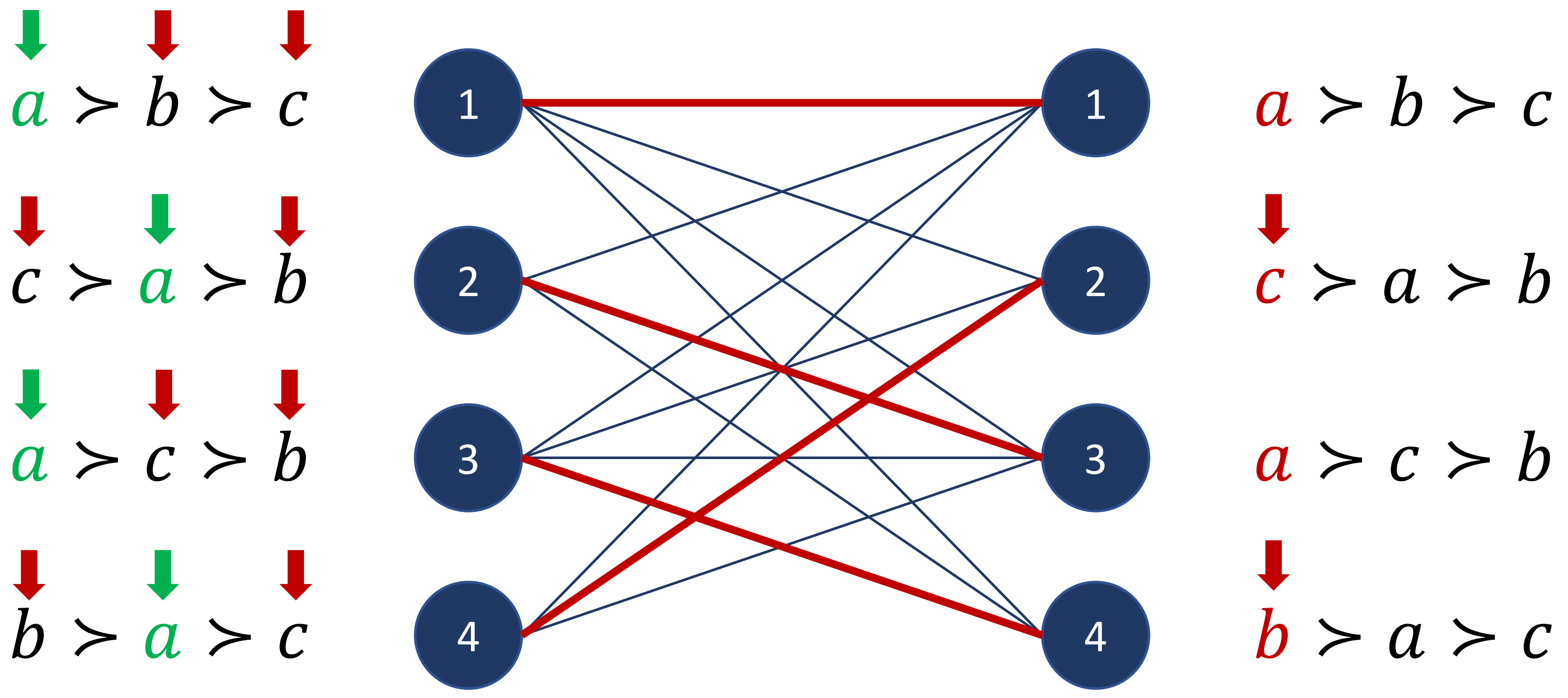
- Theorem [Kempe 2020a]:
  - The distortion of ranked pairs and Schulze's rule is $\Theta(\sqrt{m})$.
  - Analysis via a powerful LP duality approach
- Theorem [Munagala, Wang, 2019]:
  - There exists a deterministic voting rule with distortion $2 + \sqrt{5} \approx 4.236$.
- Theorem [Gkatzelis, Halpern, Shah, 2020]:
  - There exists a deterministic voting rule, PluralityMatching, with distortion 3.
  - Proof by confirming a conjecture by [Munagala, Wang, 2019]
- Theorem [Kizilkaya, Kempe, 2022]:
  - There exists a deterministic voting rule, Plurality Veto, with distortion 3.
  - Proof by confirming a conjecture by [Munagala, Wang, 2019] in a 1-paragraph proof

Domination Graph of Candidate a

Certificate that a is a good choice:

we can match each voter j (with top choice x) to another voter $i = M(j)$ with $a \succcurlyeq_i x$.

Edge (i, j) exists when, in i's vote, a weakly defeats the top choice of j



Perfect Matching

Perfect Matching Gives Distortion 3

- Lemma [Munagala, Wang, 2019; Kempe 2020a]
  - If the domination graph of a has a perfect matching, then a has distortion at most 3.
  - Conjecture: For every profile, at least one candidate's graph has a perfect matching.

- Proof (skip):

$$
\begin{aligned}
\mathrm{SC}(a) &= \sum_{i \in V} d(i,a) \\
&\le \sum_{i \in V} d(i, \mathrm{top}(M(i))) && (\because a \succcurlyeq_i \mathrm{top}(M(i)), \forall i \in V) \\
&\le \sum_{i \in V} (d(i,b) + d(b, \mathrm{top}(M(i)))) && (\because \text{triangle inequality}) \\
&= \sum_{i \in V} (d(i,b) + d(b, \mathrm{top}(i))) && (\because M \text{ is a perfect matching}) \\
&\le \sum_{i \in V} (d(i,b) + d(b,i) + d(i, \mathrm{top}(i))) && (\because \text{triangle inequality}) \\
&\le \sum_{i \in V} (d(i,b) + d(b,i) + d(i,b)) \\
&= 3 \cdot \mathrm{SC}(b).
\end{aligned}
$$

Plurality Veto

- Simple voting rule that selects a candidate with a perfect matching in the domination graph. [Kizilkaya, Kempe, 2022]
  - All alternatives start out being alive. Each voter i gives 1 point to i's top alternative.
  - Go through voters 1-by-1 in an arbitrary order.
  - Each voter i subtracts 1 point from i's least-favorite alive alternative.
    If that alternative's score drops to 0, it dies.
  - The alternative a surviving until the last round wins.
- Only two queries per voter!
- Note: there are n points in total, and we take n points away.
- In the domination graph of a:
  - For each x, we can match the t voters who rank x top with the t voters who delete a point from x during the execution of the rule.
  - For each such voter, $a \succcurlyeq_i x$ because a is alive.
- Can make it anonymous and neutral via "eating" / "reverse Phragmén" [Kizilkaya, Kempe, 2022; Peters 2023]

Randomized Rules

- Theorem [Anshelevich, Bhardwaj, Elkind, Postl, Skowron, 2018]:
  - No randomized rule has distortion better than 2.
    - Same example as before
  - Random Dictatorship has distortion $3 - {}^{2}/_{n}$.

- Theorem [Kempe 2020a]:
  - There is a randomized voting rule with access only to top choices with distortion $3 - {}^{2}/_{m}$.

- Theorem [Charikar, Ramakrishnan, 2022; Pulyassary, Swamy, 2021]:
  - No randomized rule has distortion better than 2.1126 for all m.

- Theorem [Charikar, Ramakrishnan, Wang, Wu 2023]:
  - A mixture between maximal lottery and random dictatorship on a subset of alternatives gets 2.753

- Open question: What is the optimal metric distortion of randomized rules?
- Open question: Is the instance-optimal randomized rule polytime computable?

Many, Many Open Questions

- Extensions for metric distortion less-studied than for utilitarian distortion
  - Participatory budgeting?
  - Strategyproofness?
  - Ranked ballots + additional queries?
  - Information-distortion tradeoff? [Kempe 2020a]
  - …

Outline

- Introduction
  - Applications of voting
  - Motivating the distortion framework
- Utilitarian distortion framework
  - Model
  - Known results
- Metric distortion framework
  - Model
  - Known results
- Applications beyond voting

Actually, More Voting First!

- Distributed elections
  - Voters partitioned into groups that conduct separate elections [Borodin, Lev, Shah, Strangway, 2019; Filos-Ratsikas, Micha, Voudouris, 2020; Filos-Ratsikas, Voudouris, 2021; Anshelevich, Filos-Ratsikas, Voudouris, 2022]
- Representative candidates
  - Alternatives sampled from the pool of voters [Cheng, Dughmi, Kempe, 2017; Cheng, Dughmi, Kempe, 2018]
- Voter abstentions
  - What if only a fraction of the voters vote? [Borodin, Lev, Shah, Strangway, 2019; Seddighin, Latifian, Ghodsi, 2021; Anagnostides, Fotakis, Patsilinakos, 2021]
- Approval-based cost functions for metric distortion [Pierczynski, Skowron, 2019]

Beyond Voting

- One-Sided Matching
  - Match m agents to m items, where agents have cardinal utilities for the items but only provide ordinal rankings
- Theorem [Filos-Ratsikas, Frederiksen, Zhang, 2014]:
  - The best distortion of any randomized rule is $\Theta(\sqrt{m})$.
- Theorem [Amanatidis, Birmpas, Filos-Ratsikas, Voudouris, 2021]:
  - The best distortion of any deterministic rule is $\Theta(m^2)$.
  - They also analyze the information-distortion tradeoff via queries.
- Surprisingly, identical bounds as single-winner voting!
- Other work [Ma, Menon, Larson, 2021; Bishop, Chan, Mandal, Tran-Thanh, 2022]

Beyond Voting

- Resource allocation
  - Allocate m goods to n agents
  - [Halpern, Shah, 2021]: When every agent ranks the goods
  - [Ebadian, Freeman, Shah, 2022]: When k agents provide no information while the rest provide cardinal utilities
- Secretary problem [Hoefer, Kodric, 2017]
- Graph-theoretic problems
  - Maximum-weight matching [Anshelevich, Sekar, 2016a]
  - Max k-sum, densest k-subgraph, maximum traveling salesman [Anshelevich, Sekar, 2016b]
  - Min-weight and max-min bipartite matching, facility location, k-center, k-median [Filos-Ratsikas, Voudouris, 2021; Anshelevich, Zhu, 2021]

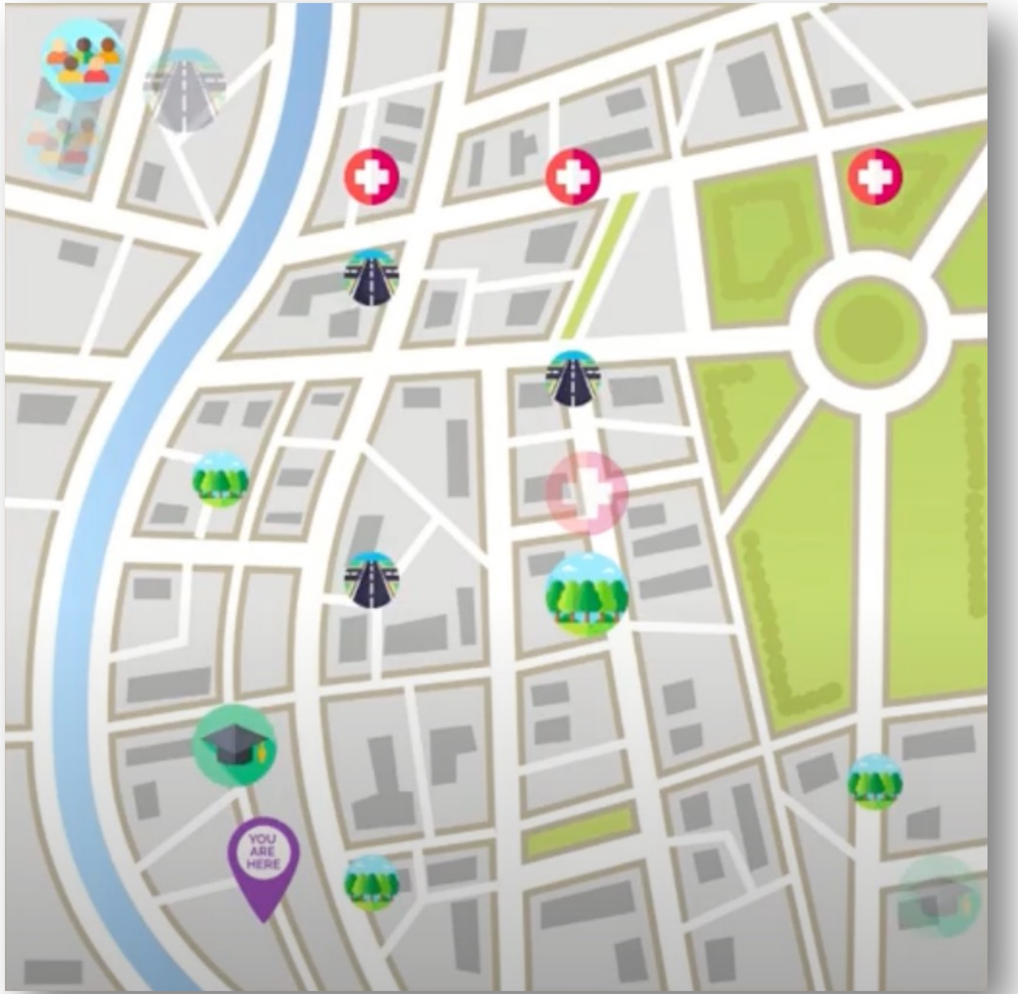
Future Work: Ballot Design

- Common ballot designs
  - Pairwise comparisons, “Do you like candidate a at least twice as much as candidate b?”, …
- Better models of cognitive burden
  - Psychology, HCI, …
- Voter errors in answering ballots
  - Expressive ballots can also induce errors
- Intangible aspects of ballot design
  - Barcelona PB team: “Knapsack votes are good because they help voters understand the limitations of the budget.”

Future Work: Distortion vs Other Desiderata

- Distortion & Truthfulness
  - With ranked ballots, near-optimal distortion can be achieved via truthful aggregation
  - What happens with other ballot formats?
- Distortion & Axioms
  - Can we achieve low distortion together with popular axioms?
  - Especially, proportional representation for committee selection
- Distortion & Explainability
  - Explaining the voting rule vs explaining what it does

Future Work: More Complex Voting Paradigms



- Design optimal voting rules for more complex voting paradigms
  - Participatory budgeting
  - Districting
- Model end-to-end voting
  - In participatory budgeting, voting is but the final step of a year-long process
- Compare models of democracy
  - E.g., direct democracy, representative democracy, and liquid democracy

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Thank you!

Questions?