Optimized Democracy

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Approval Voting

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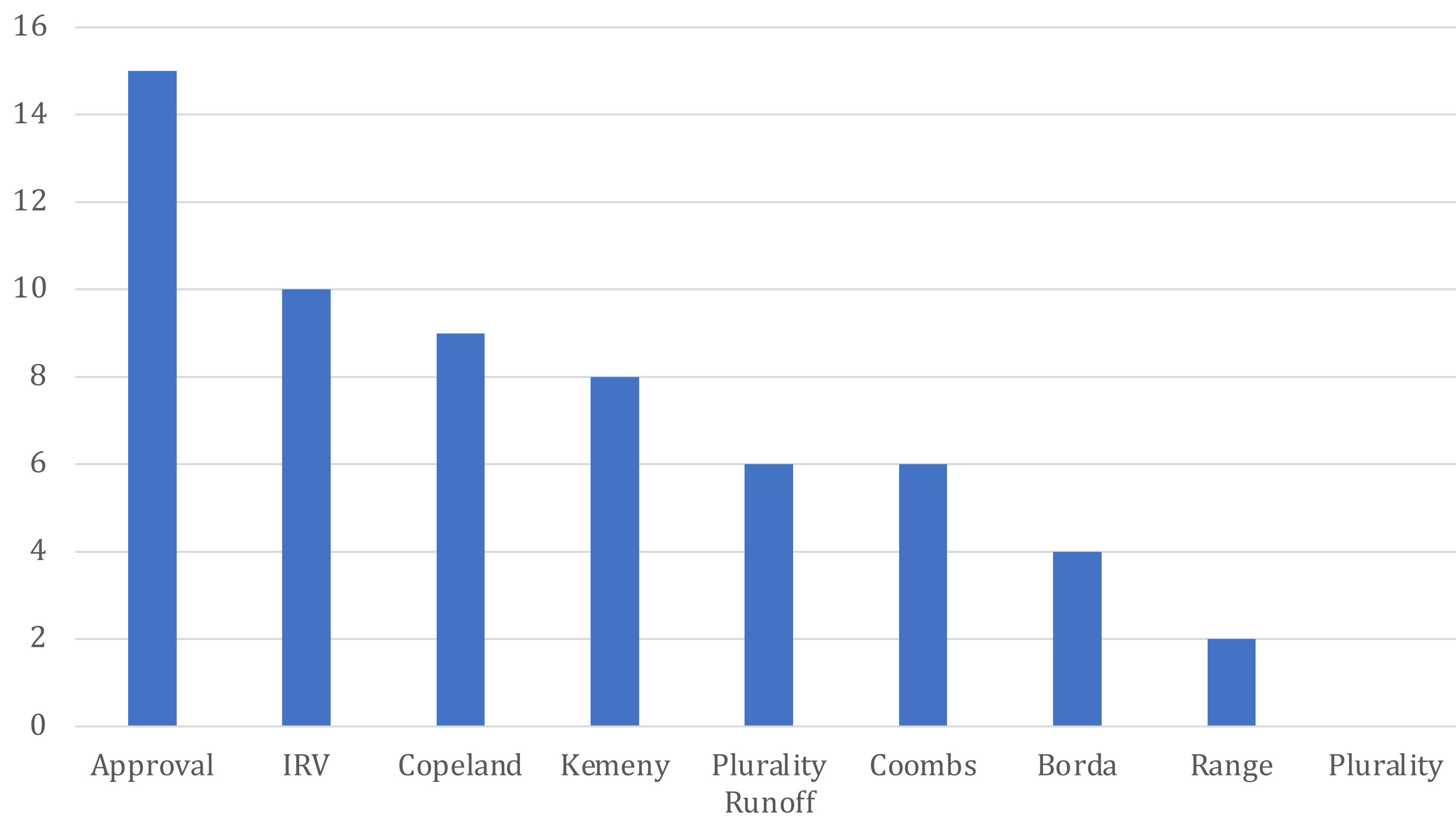
Approval Voting

“vote for as many as you like”

Approval Voting (AV) elects an alternatives that is approved by the highest number of voters

Asking voting theorists: “What is the best voting rule for your town to use to elect the mayor?"



(They used approval voting to vote.)

Reasons Election Reform Advocates Give For AV

- More expressive than plurality, simpler than rankings
- Reduces spoiler effect
- Results are easy to understand
- Non-frontrunners get a more accurate measure of support

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Results from an Instant Runoff Election

Full distribution of preferences

	HALL Tom (GRN)		HART Ross (ALP)		WOODBURY Susan (AJP)		ARCHER Bridget (LP)		LAMBERT Todd (IND)		ROARK Allan John (UAPP)		COOPER Carl (NP)		
Count	**Votes**	**%**	**Votes**	**%**	**Votes**	**%**	**Votes**	**%**	**Votes**	**%**	**Votes**	**%**	**Votes**	**%**	**Total**
First	7,202	10.48	23,878	34.74	1,667	2.43	29,094	42.33	2,607	3.79	3,342	4.86	943	1.37	68,733
Second	55	5.83	73	7.74	38	4.03	342	36.27	186	19.72	249	26.41	**Excluded**		943
Total	7,257	10.56	23,951	34.85	1,705	2.48	29,436	42.83	2,793	4.06	3,591	5.22			68,733
Third	466	27.33	357	20.94	**Excluded**		275	16.13	419	24.57	188	11.03			1,705
Total	7,723	11.24	24,308	35.37			29,711	43.23	3,212	4.67	3,779	5.50			68,733
Fourth	469	14.60	767	23.88			781	24.32	**Excluded**		1,195	37.20			3,212
Total	8,192	11.92	25,075	36.48			30,492	44.36			4,974	7.24			68,733
Fifth	858	17.25	1,449	29.13			2,667	53.62			**Excluded**				4,974
Total	9,050	13.17	26,524	38.59			33,159	48.24							68,733
Sixth	**Excluded**		7,561	83.55			1,489	16.45							9,050
Total			34,085	49.59			34,648	50.41							68,733
							Elected								

(Australian House of Representatives, 2019, Bass, TAS)

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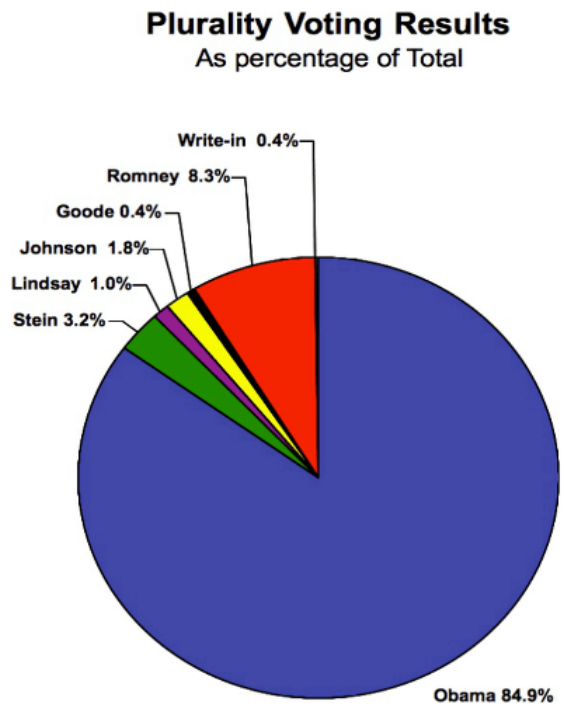
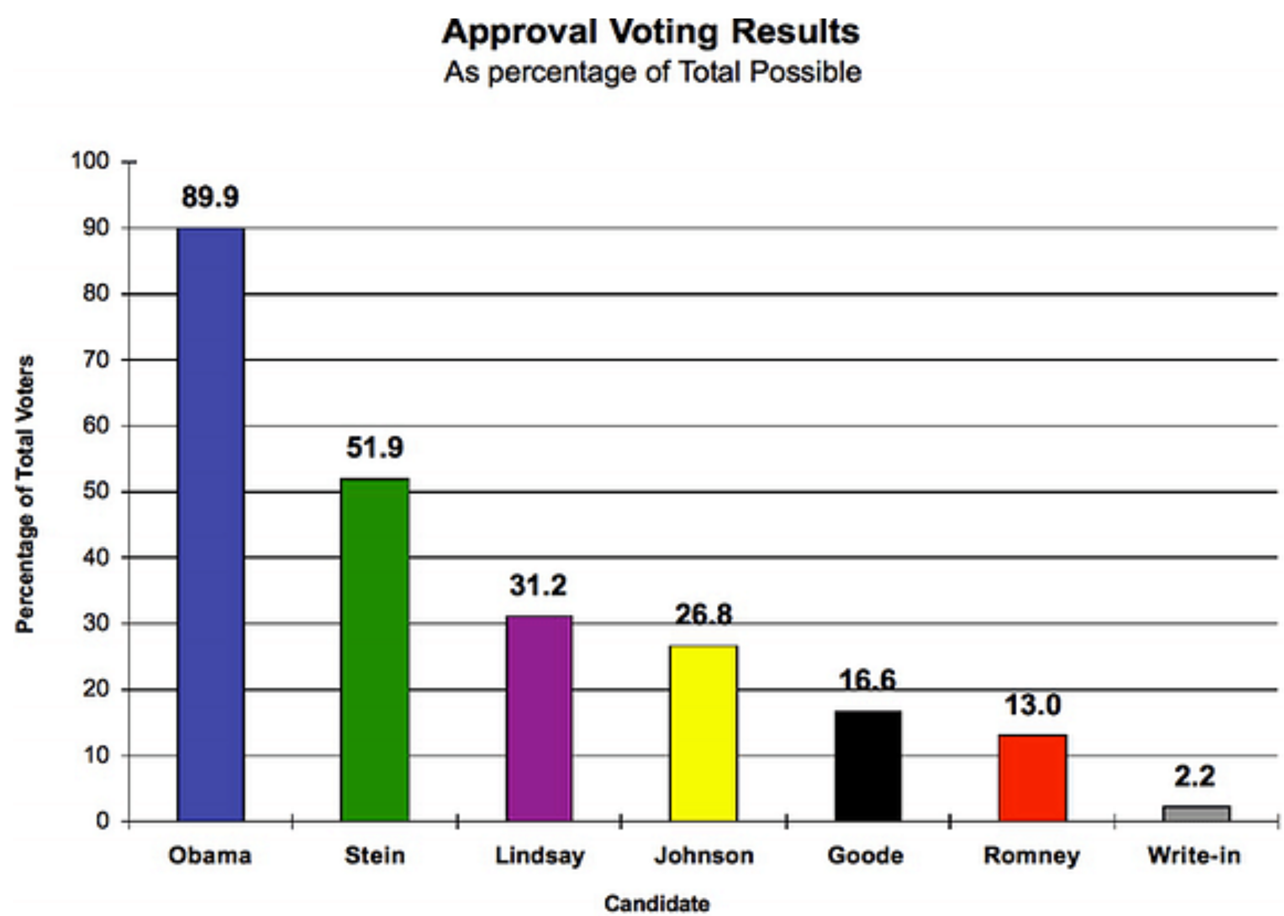
An ”Exit Poll” in Manhattan, 2008





Dichotomous Preferences

- An easy way to analyze AV formally is to assume voters have *dichotomous preferences:*
  - They strictly prefer approved alternatives to disapproved alternatives
  - They are indifferent between all approved alternatives
  - They are indifferent between all disapproved alternatives

AV and Condorcet

- Assuming dichotomous preferences, every approval winner x is a weak Condorcet winner, in the sense that for all alternatives y, at least half of the voters weakly prefer x to y.
- Proof: Assume not. Then a strict majority of voters strictly prefers y to x, and thus all these voters approve y but not x. So the approval score of y is strictly higher than $\frac{n}{2}$, and x is strictly below $\frac{n}{2}$, contradicting that x is an approval winner.

AV and Borda

- Assuming dichotomous preferences, approval winners and Borda winners are the same.
- This is true for all natural generalizations of Borda's rule to preferences with ties.

AV is strategyproof

- **Theorem.** Let $P: N \to 2^A$ be an approval profile, and let P' be another profile with $P(j) = P'(j)$ for all $j \in N \setminus \{i\}$.
  Then $AV(P') \cap P(i) \subseteq AV(P) \cap P(i)$.
- Proof: If not, there is $x \in AV(P') \setminus AV(P)$ with $x \in P(i)$. Take any $y \in AV(P)$. Note that $score'(x) \leq score(x) < score(y) \leq score'(y)$. This contradicts $x \in AV(P')$.
- Also: $AV(P') \cap (A \backslash P(i)) \supseteq AV(P) \cap (A \backslash P(i))$.
- Note: G-S bites again for trichotomous voters.

AV avoids spoilers

- AV is cloneproof: if we copy a winning alternative x (and voters approve the copy iff they approve x) then x and its copy still win. If we copy a losing alternative, both original and copy still lose, and the set of winners stays the same.
- AV is independent of losers: if we delete an alternative that lost, the set of winners doesn’t change.

AV without dichotomous prefs

- Without assuming dichotomous preferences, we cannot give many guarantees about AV.
- Suppose voter preferences are strict (no indifferences), and all voters vote sincerely (they approve a prefix.

$$\boxed{a \succ b \succ c \succ d} \succ e, \qquad \boxed{b \succ d} \succ e \succ a \succ c$$

Question

1. Which alternatives can be made approval winners with sincere voting?

2. Which alternatives can be made *unique* approval winners with sincere voting?

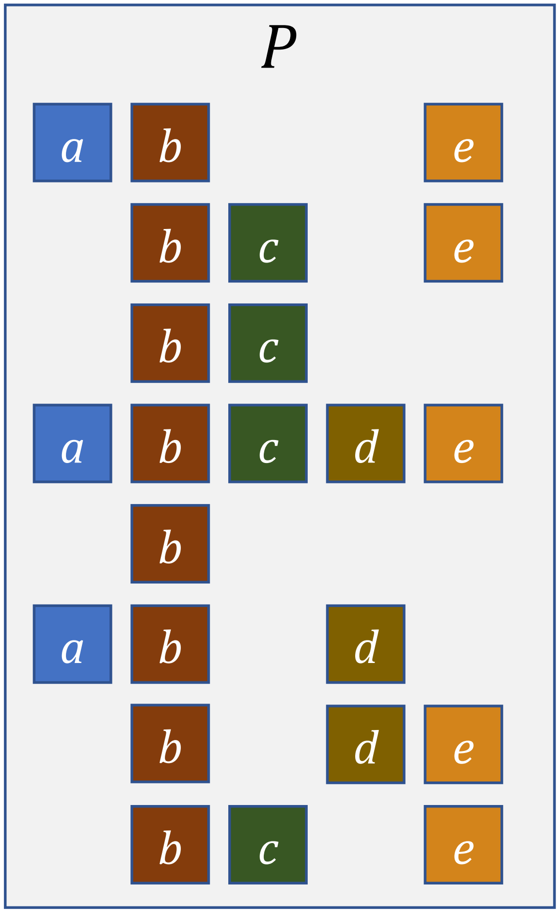
Axiomatic Characterization

Theorem.

Approval Voting is the only approval-based voting rule f (which is allowed to report ties) that satisfies:

- *Reinforcement*
  If $P_1: N \to 2^A$ and $P_2: N' \to 2^A$ are profiles defined on disjoint sets of voters, we have $f(P_1 + P_2) = f(P_1) \cap f(P_2)$ whenever $f(P_1) \cap f(P_2) \neq \emptyset$.
- *Faithfulness*
  If $P: \{i\} \to 2^A$ is a single-voter profile, then $f(P) = P(i)$.
- *Disjoint Equality*
  If $P: \{i, j\} \to 2^A$ is a two-voter profile with $P(i) \cap P(j) = \emptyset$, then $f(P) = P(i) \cup P(j)$.

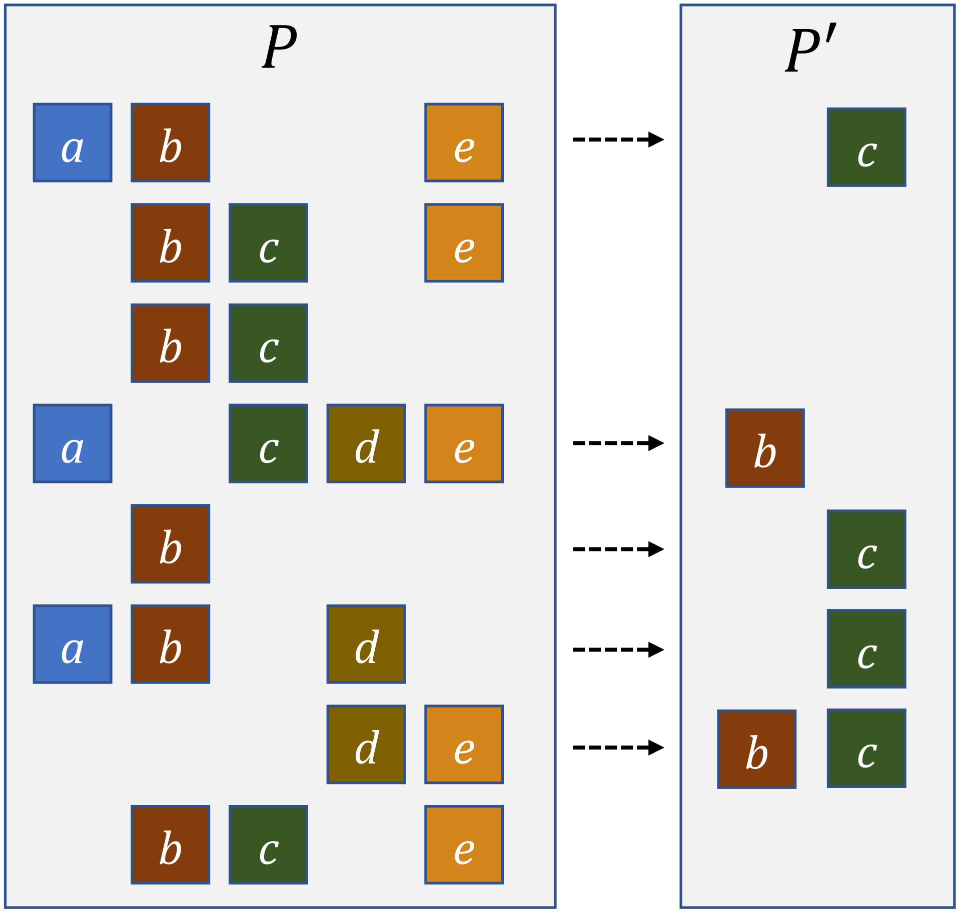
Proof.



If there is an alternative b that everyone approves, then by faithfulness and reinforcement, f selects exactly those alternatives approved by everyone.

Thus, $f(P) = AV(P)$.

Suppose there is an alternative $b \in AV(P) \setminus f(P)$.
Let $c \in f(P)$ be any alternative selected by f.
Now add some new voters:

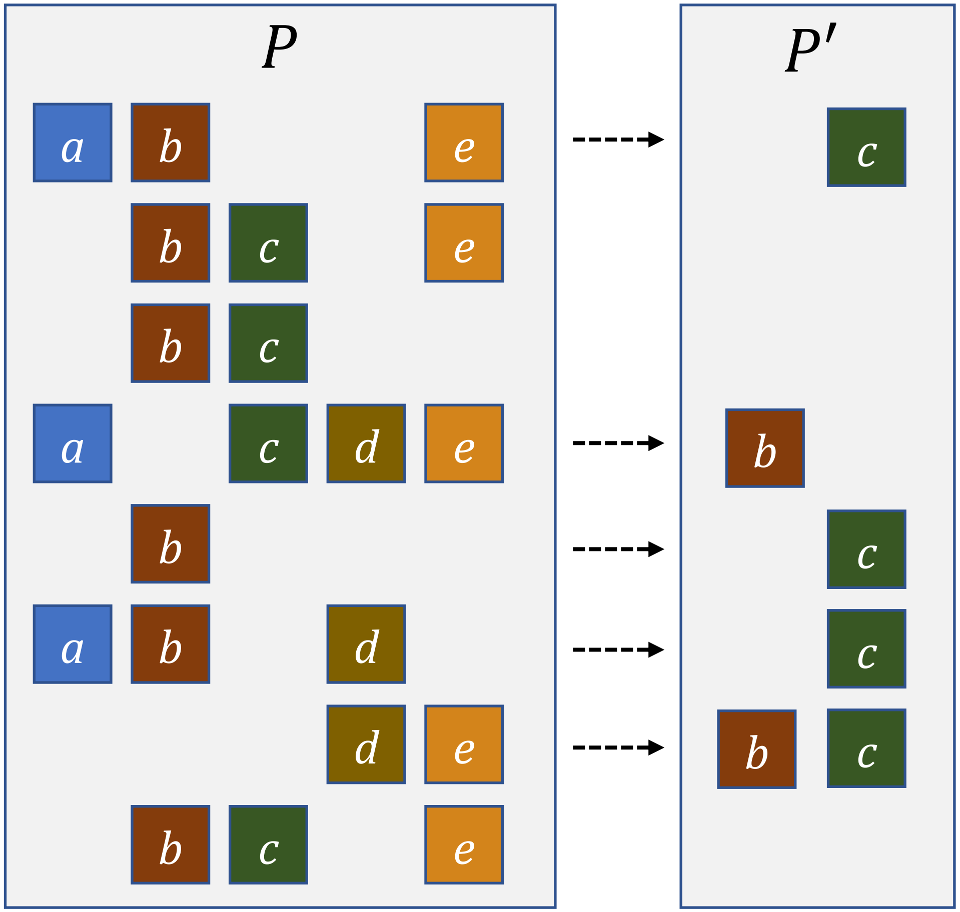


1.Pair the voters in each row. By disjoint equality, both b and c are elected. If a row has one voter, by faithfulness both b and c are elected. By reinforcement, $b, c \in f(P + P')$.

2. Since b is an approval winner, there are weakly more $\{c\}$ voters than $\{b\}$ voters in P'. So we can pair each $\{c\}$ voter with a $\{b\}$. Disjoint equality for the paired voters, faithfulness for $\{c\}$ and $\{b, c\}$ voters, and reinforcement implies that $c \in f(P')$. Also $c \in f(P)$. Since $b \notin f(P)$, we have $b \notin f(P + P')$ by reinforcement. Contradiction.

$\Rightarrow$ all approval winners are elected by f

Suppose there is an alternative $c \in f(P) \setminus AV(P)$.
Let $b \in AV(P)$ be any approval winner.
Now add some new voters:



1. As before, by pairing the voters in each row, we get $b, c \in f(P + P')$.

2. Since c is not an approval winner, there are strictly more $\{c\}$ voters than $\{b\}$ voters in P'. So we can pair each $\{c\}$ voter with a $\{b\}$, and are left with at least one additional $\{c\}$ voter. Disjoint equality for the paired voters, faithfulness for $\{c\}$ and $\{b, c\}$ voters, and reinforcement implies $f(P') = \{c\}$. Also $c \in f(P)$. Hence $f(P + P') = \{c\}$ by reinforcement. Contradiction.

$\Rightarrow$ only approval winners are elected by f $\square$

Additional Characterizations

- AV is the only rule that satisfies anonymity, neutrality, reinforcement, faithfulness, and
  - Disjoint equality (as we have seen)
  - Strategyproofness
  - Cloneproofness
  - Independence of losers
- Related Characterizations are known for ranking-based voting rules, including Borda and Plurality

Bibliography

- Brams, Steven, and Peter C. Fishburn. *Approval voting*. Springer, 2007.
- Fishburn, Peter C. "Axioms for approval voting: Direct proof." *Journal of Economic Theory* 19.1 (1978): 180-185.
- Brandl, Florian, and Dominik Peters "Simple Characterizations of Approval Voting", 2019.